Design of Light-Tree Based Logical Topologies for Multicast Streams in Wavelength Routed Optical Networks

De-Nian Yang and Wanjiun Liao
Department of Electrical Engineering
National Taiwan University
Taipei, Taiwan
wjliao@cc.ee.ntu.edu.tw

Abstract*—In this paper, we formulate an optimization problem for the design of light-tree based logical topology in Wavelength Division Multiplexing (WDM) networks. The problem is comprised of two parts: (1) multicast routing and wavelength assignment of light-trees, and (2) the design of light-tree based logical topology for multicast streams. In the first part, we use Mixed Integer Linear Programming (MILP) to solve the optimal routing and wavelength assignment problem of light-trees with an end-to-end delay bound, and obtain the optimal placement of power splitters and wavelength converters. The numerical results show that networks with just a few power splitters and wavelength converters can efficiently carry multicast data. In the second part, we extend the above formulation to design the logical topology based on light-trees for multicast streams. In our approach, a light-tree can carry data of multiple multicast streams, and data of a multicast stream may traverse multiple light-trees to reach a receiver. The numerical results show that our approach use network resources more efficiently, as compared to the approach with a separate light-tree for a multicast stream and to the approach of transporting multicast streams over lightpath based logical networks.

Keywords- Mixed Integer linear programming (MILP); WDM; multicas; light tree; logical topology

I. INTRODUCTION

Optical networks based on Wavelength Division Multiplexing (WDM) are the most promising candidates for the next generation backbone networks. In a wavelength-routed network, data are transported in all-optical WDM channels referred to as lightpaths. The set of lightpaths forms the logical topology of a WDM network. Data are processed electronically only at each node of a logical topology (i.e., at the endpoints of the lightpath), and switched optically at intermediate nodes of the underlying physical network. Design of logical topologies has been studied in recent years [1-4], and the goal is to minimize the congestion (i.e., the load on each channel), or the resources used in physical networks.

Multicast is an efficient way to transport the data of one-to-many and many-to-many applications. Supporting multicast in logical networks is less efficient than physical networks since the latter provides higher connectivity. [5-10]. To support multicast in the WDM layer, Sahasrabuddhe and Mukherjee [6] introduce the concept of the light-tree, which is a point-to-multipoint extension of a lightpath. Branching nodes of a light-tree are equipped with *power splitters*. It is widely believed and desired that only a portion of the network nodes are required to be equipped with power splitters due to their expensive cost. This kind of networks is called a *sparse-splitting* network. In comparison with multicasting in logical networks, transporting multicast streams on light-trees uses less number of wavelength channels [1], optical transmitters, and receivers [6]. However, transporting each multicast stream on a different light-tree is also inefficient if the data rate of a stream is much lower than the data rate provided by a wavelength channel. In this case, it is more efficient to multiplex multiple multicast streams on a light-tree. Since the senders and receivers of different streams may be different, a multicast stream may need to be transmitted on multiple light-trees to reach all the receivers. In other words, data of a multicast stream may need to traverse multiple light-trees to reach each receiver. To provide such service efficiently, the following three issues need to be addressed: (1) to determine the destinations of each light-tree, (2) to determine the routing and wavelength assignment of each light tree, and (3) to determine the set of light-trees used by a multicast stream, provided that the set of multicast streams to be transmitted is given. Interestingly, this problem is just equivalent to designing a light-tree based logical topology for multicast streams. The light-tree based logical topology is a *hypergraph* [10], in which each link of the logical topology is a *hyperedge*. Here a hyperedge is a link by which more than two nodes can be connected. Each hyperedge represents a light-tree in WDM networks.

Mixed Integer Linear Programming (MILP) is a popular technique used to solve optimization problems. Previous work based on this technique for WDM networks mainly focus on the following problems:

- Routing of light-trees. Optimal routing for multicast streams in packet networks has been discussed in [11]. However, this formulation cannot be applied directly to WDM networks because it assumes that every node is

[1] A wavelength channel means an edge on a light-tree.

capable of multicasting. The formulation in [12,13] obtains the optimal routing for light-trees in sparse-splitting optical networks. However, the number of variables or constraints grows exponentially with the network size, i.e., the number of nodes and links in a network. As a result, their formulation is not practical for large networks. The formulation in [14] uses less number of variables and constraints for the same problem. Note that [12-14] do not consider wavelength assignment in the network. Only the routing problem is discussed.

- Design of logical topologies. Formulation to design the light-path based logical topologies for unicast streams are provided in [1-4]. Mellia et al. [15] proposes a formulation to design the logical topologies for multicast streams. Both approaches use lightpaths, instead of light-trees, to form the logical topologies. Moreover, they assume that either all nodes are capable of wavelength conversion, or no node is capable of wavelength conversion.
- Optimal placement of power splitters and wavelength converters. Both power splitters and wavelength converters are considered scarce resource, and should be allocated carefully [13,16-18]. Ali and Deogun [13] formulate an optimization problem to allocate power splitters in WDM networks. However, this formulation is not scalable because the number of variables and constraints again grows exponentially with the network size. Subramaniam et al. [16,18] propose an analytical method to analysis the blocking probability of unicast traffic with different number of wavelength converters in physical networks. A dynamic programming method is proposed to find the optimal placement of wavelength converters. Xiao et al. [17] incorporates Linear Programming (LP) formulation into their algorithm to determine where to place wavelength converters in physical networks for unicast traffic.

Light-tree based logical topologies have been studied in [19,20] using heuristics. A special kind of hypergraph, Kautz hypergraph, is used to design the logical topologies. On routing and wavelength assignment of a hyperedge, the authors assume that each physical node is capable of both wavelength conversion and power splitting.

In this paper, we study the design of optimal light-trees based logical networks for multicast streams in WDM networks. The problem is comprised of two parts: (1) multicast routing and wavelength assignment of light-trees and (2) the design of light-tree based logical topology for multicast streams. In the first part, we use MILP to solve the multicast routing and wavelength assignment of light-trees. The number of nodes capable of power splitting or wavelength conversion is given as a parameter (i.e., known *a priori*). Our formulation has the following characteristics:

- End-to-end delay of a light-tree is bounded.
- Optimal placement of wavelength converters and power splitters is obtained.

In the second part, we extend the above formulation to design the logical topology based on light-trees for multicast streams. The light-tree based logical topology is a hypergraph. Our formulation has the following characteristics:

- Data carried on the logical topology are multicast streams.
- The end-to-end delay of each multicast stream is bounded.
- The routing of each multicast stream in the logical topology is obtained.

A common problem of MILP formulation is the difficulty in requiring some constraints to hold only under a certain condition. For example, the flow conservation rule states that the amount of flows incoming to a node is the same as the amount of flows outgoing from the node. This rule holds for a node only if the node is not capable of power splitting. It is hard to make the constraint *linear* if the capability of a node is to be decided (e.g., whether a node should be equipped with power splitters or wavelength converters). To solve this problem, we incorporate a large number, say, M into the formulation. In our approach, for the node which does not follow the flow conservation rule, M will force the constraint to always hold. This technique enables MILP to formulate more complicated scenarios for WDM networks, while the number of variables and constraints only increases slightly. We will demonstrate how to use this technique in the next section.

The rest of the paper is organized as follows. In Section II, our formulation is presented and explained in detail. The formulation to solve the routing and wavelength assignment of light-trees is presented first. Then, we extend the formulation to solve the logical topology design problem. In Section III, numerical results obtained by our formulation for an example network are shown. In Section IV, we discuss the complexity of our formulation. Finally, we conclude this paper in Section V.

II. PROBLEM FORMULATION

The problem to be studied is comprised of two parts: routing and wavelength assignment of light-trees, and logical topology design. We assume the physical topology, the number of power splitters and wavelength converters in the network, and the number of optical transmitters and receivers at each node are given as parameters. We assume that the power loss of power splitting can be neglected because optical amplifiers are used. We also assume that each wavelength converter is a *full range wavelength converter* [16], i.e., each wavelength on a channel can be converted to any other wavelength.

The notation is defined as follows.

V	the set of all nodes in a network;
E	the set of all links in a network;
Γ	the set of light-trees to be created;
H	the set of multicast streams;
Λ	the set of wavelengths supported in a fiber;
$In(n)$	the set of links incident to node n, i.e., n is the sink of the links, $n \in V$;

$Out(n)$	the set of links incident from node n, i.e., n is the source of the links, $n \in V$;
T_n	the set of light-trees rooted at node n, $n \in V$;
$Up(e)$	the transmitting node of link e, $e \in E$;
$Dn(e)$	the receiving node of link e, $e \in E$;
k	a wavelength in a fiber, $k \in \Lambda$;
t	a light tree, $t \in \Gamma$.

Multiple fibers on a link are supported in our formulation. We assume that the set of wavelengths that a fiber can support is the same for all fibers. There is one *source* and multiple *destinations* on a light-tree. We define an *incoming channel* of a node n as a wavelength channel of a fiber on a link which belongs to $In(n)$. Similarly, an *outgoing channel* of n is a wavelength channel of a fiber on a link belonging to $Out(n)$. We also define the *outgoing wavelengths* of n used by light-tree t as the wavelengths assigned to outgoing channels of n which are used by t. For a light-tree, we call m an *upstream node* of n if there is a path from m to n on the light-tree. In the above case, we can also call n a *downstream node* of m.

A. Routing and Wavelength Assignment of Light-trees

We use MILP to find (1) the optimal routing and wavelength assignment of light-trees with an end-to-end delay bound, and (2) the optimal placement of power splitters and wavelength converters in a WDM network. The parameters which are given as a part of problems are denoted as follows.

N_S	the number of power splitters in a network;
N_C	the number of wavelength converters in a network;
Rx_n	the number of optical receivers at node n;
Tx_n	the number of optical transmitters at node n;
F_e	the number of fibers on link e;
P_e	the propagation delay of link e; we assume $P_e > 0$ for $\forall e \in E$;
PT_t	the end-to-end delay bound of light-tree t;
$D_n^{(t)}$	if node n is a destination of light-tree t, $D_n^{(t)} = 1$; otherwise, $D_n^{(t)} = 0$;
M	a constant used to enforce conditional rules to the constraints; the value of M is set as follows.

$$M = \max\left(\sum_{t\in\Gamma} PT_t, |E| \times |\Lambda| \times F_e, \sum_{e\in E} P_e\right) \quad (1)$$

We note that the number of light-trees in which a node n participates must be equal to or less than the number of optical receivers at n.

The variables to be decided are defined as follows.

$I_{e,k}^{(t)}$	the number of fibers of link e in which wavelength k is used by light-tree t. $I_{e,k}^{(t)}$ must be a positive integer or 0;
$J_e^{(t)}$	if any wavelength of a fiber on link e is used by light-tree t, $J_e^{(t)} = 1$; otherwise, $J_e^{(t)} = 0$;
$Q_n^{(t)}$	the end-to-end delay from the source of light-tree t to node n, $Q_n^{(t)} \geq 0$;
Φ_n	if node n is capable of power splitting, $\Phi_n = 1$; otherwise, $\Phi_n = 0$;
Θ_n	if node n is capable of wavelength conversion, $\Theta_n = 1$; otherwise, $\Theta_n = 0$;
$G_k^{(t)}$	if light-tree t uses wavelength k of any fiber incident from the source of t, $G_k^{(t)} = 1$; otherwise, $G_k^{(t)} = 0$

The objective function is formulated as follows.

$$\text{minimize} \sum_{t\in\Gamma}\sum_{e\in E}\sum_{k\in\Lambda} I_{e,k}^{(t)} \quad (2)$$

Similar to [14], this objective function is to minimize the resource, i.e. the number of wavelength channels, used by all light-trees in a WDM networks. The goal of most existing work [5-9] is to maximize the number of light-trees which can be created in the network. Note that our objective function is highly correlated with this goal because more light-trees can be created as the resource used by all light-trees decreases. Moreover, some other objective functions proposed in existing works can be used as the objective function in our formulation. For example, with some minor modification to our formulation, the following two objective functions can be achieved: (1) to minimize the number of optical transmitters and receivers in a network, i.e., the objective function in [6] and (2) to minimize the overall profit of creating light-trees, i.e., the objective function in [12,13].

The set of constraints is shown as follows..

1) Routing and wavelength assignment constraints:

$$\sum_{t\in\Gamma} I_{e,k}^{(t)} \leq F_e, \quad \forall e \in E, \forall k \in \Lambda \quad (3)$$

$$\frac{1}{M} \times \sum_{k\in\Lambda} I_{e,k}^{(t)} \leq J_e^{(t)}, \quad \forall e \in E, \forall t \in \Gamma \quad (4)$$

$$\frac{D_n^{(t)} + \sum_{e\in Out(n)}\sum_{k\in\Lambda} I_{e,k}^{(t)}}{M} \leq \sum_{e\in In(n)}\sum_{k\in\Lambda} I_{e,k}^{(t)}, \ \forall n \in V, \forall t \in \Gamma, t \notin T_n \quad (5)$$

In the conventional multi-commodity flow assignment problem, a *flow conservation constraint* is required to ensure that the amount of outgoing flows of a node must be equal to the amount of incoming flows if the node is neither the source nor a destination. In optical multicast networks, this constraint does not hold because the flow, i.e., optical signal, can be duplicated. We use eqs. (3)-(5) to make sure that there is a path from the source to each destination on the light-tree. Eq. (3) ensures that each wavelength channel of a link is used by at most F_e light-trees. Eq. (4) guarantees that $J_e^{(t)}$ equals one if at least one wavelength channel of a fiber on link e is used by light-tree t. If node n is not the source of t and at least one of the following conditions holds,

- n is a destination of light-tree t
- at least one outgoing channel of n is used by t

then eq. (5) guarantees that t uses at least one incoming channel of n. Since the incoming channel of n must be an outgoing channel of another node, say, m, eq. (5) also ensures that at least one incoming channel of m is used by t if m is not the source of t. Therefore, there must exist a path from the source of t to n, or n and m are in a loop, i.e., n is also an upstream node of m. The delay constraints specified later will guarantee that there is no loop in any feasible solution.

2) Node constraints:

$$\sum_{e\in Out(n)}\sum_{k\in\Lambda} I_{e,k}^{(t)} + D_n^{(t)} \le (\Phi_n)\times M + \sum_{e\in In(n)}\sum_{k\in\Lambda} I_{e,k}^{(t)},\quad \forall n\in V, \forall t\in\Gamma, t\notin T_n \tag{6}$$

$$\sum_{e\in Out(n)} I_{e,k}^{(t)} \le (\Phi_n + \Theta_n)\times M + \sum_{e\in In(n)} I_{e,k}^{(t)},\quad \forall n\in V, \forall t\in\Gamma, t\notin T_n, \forall k\in\Lambda \tag{7}$$

$$\sum_{e\in In(n)} I_{e,k}^{(t)} \ge \frac{\sum_{e\in Out(n)} I_{e,k}^{(t)}}{M} - \Theta_n, \forall n\in V, \forall t\in T_n, t\notin T_n, \forall k\in\Lambda \tag{8}$$

$$\sum_{n\in V}\Phi_n \le N_S \tag{9}$$

$$\sum_{n\in V}\Theta_n \le N_C \tag{10}$$

Eqs. (6)-(10) ensure that the result will not contradict with the capability of a node. We explain in detail the above constraints in the following four cases:

- $\Phi_n = 0$ and $\Theta_n = 0$: the wavelength assigned to an incoming channel can be neither converted to any other wavelength for an outgoing channel nor split into multiple outgoing channels. For all incoming/outgoing channels of node n using wavelength k, eq. (7) guarantees that for node n, the number of incoming channels used by light-tree t must be equal to or larger than the number of outgoing channels used by t if n is not the source of t. Eq. (6) states that node n needs one more incoming channel if n is a destination of t. Eq. (8) holds if eq. (7) is satisfied.
- $\Phi_n = 0$ and $\Theta_n = 1$: the wavelength assigned to an incoming channel can be converted to any other wavelength but cannot be split to multiple channels. Because node n is incapable of power splitting, eq. (6) ensures that the number of incoming channels of n used by t must be equal to or larger than the number of outgoing channels of n used by t. One more incoming channel is needed if n is a destination of t. Since node n is capable of wavelength conversion, eqs. (7) and (8) are unnecessary. Here M makes eqs. (7) and (8) always hold for any assignment of values to the variables.
- $\Phi_n = 1$ and $\Theta_n = 0$: the wavelength assigned to an incoming channel can be split but cannot be converted. For all incoming/outgoing channels of node n using wavelength k, eq. (8) guarantees that at least one incoming channel is used by light-tree t if any outgoing channel is used by t. Since node n is capable of power splitting, eqs. (6) and (7) are unnecessary. Here M makes eqs. (6) and (7) always hold for any assignment of values to the variables.
- $\Phi_n = 1$ and $\Theta_n = 1$: M makes eqs. (6)-(8) always hold for any assignment of values to the variables.

Eqs. (9) and (10) ensures that the number of nodes equipped with power splitters or wavelength converters must be less than or equal to N_S or N_C, both of which can be given according to the budget of a network provider.

3) Delay constraints:

$$Q_n^{(t)} = 0,\ \ \forall n\in V, \forall t\in T_n \tag{11}$$

$$1 - J_e^{(t)} - \frac{Q_{Up(e)}^{(t)} + P_e - Q_{Dn(e)}^{(t)}}{M} \ge 0, \forall e\in E, \forall t\in\Gamma \tag{12}$$

$$D_n^{(t)} + \frac{Q_n^{(t)} - PT_t}{M} \le 1, \forall n\in V, \forall t\in\Gamma \tag{13}$$

Eqs. (11)-(13) ensure that the delay of the path from the source of a light-tree to each destination must be equal to or less than the delay bound. Eq. (11) states that the delay from the source of a light-tree t to a node n equals zero if n is the source of t. Eq. (12) ensures that if link e from node m to n is used by t, then the delay from the source of t to n is equal to or larger than the delay from the source of t to m plus the propagation delay of e. If e is not used by t, i.e., $J_e^{(t)} = 0$, M makes eq. (12) always hold. Eq. (13) guarantees that the delay from the source of t to n must be equal to or less than the delay bound if n is a destination of t.

Eq. (12) also ensures that there is no loop in any feasible solution. If there is a loop and nodes m and n are both in the loop, the delays from m to n and n to m must both be zero, which contradict our assumption that the propagation delays of all links are larger than zero. With the delay constraint and the routing and wavelength constraints specified above, there must exist a path from the source of a light-tree to each destination. With the objective function, redundant paths can be pruned. Therefore the optimal solution must be a tree.

4) Optical transceiver constraints:

$$\sum_{e\in Out(n)}\sum_{t\in T_n}\sum_{k\in\Lambda} I_{e,k}^{(t)} \le Tx_n + \Phi_n \times M,\ \forall n\in V \tag{14}$$

$$\frac{\sum_{e\in Out(n)} I_{e,k}^{(t)}}{M} \le G_k^{(t)},\ \forall n\in V, \forall t\in T_n, \forall k\in\Lambda \tag{15}$$

$$\sum_{t\in T_n}\sum_{k\in\Lambda} G_k^{(t)} \le Tx_n + \Theta_n \times M,\ \forall n \in V \qquad (16)$$

$$\sum_{t\in\Gamma} D_n^{(t)} \le Rx_n,\ \forall n \in V \qquad (17)$$

Eqs. (14)-(17) ensure that the number of optical transmitters and receivers used by a light-tree must comply with the capability of the source and destinations, i.e., whether the source and destinations is capable of power splitting and wavelength conversion. We explain in detail the above constraints in the following four cases:

- $\Phi_n = 0$ and $\Theta_n = 0$: in this case, the number of optical transmitters used by all light-trees rooted at *n* is equal to the number of outgoing channels of *n* used by all light-trees. Eq. (14) guarantees that this number must be equal to or less than the number of optical transmitters at *n*. Eqs. (15) and (16) also hold if eq. (14) is satisfied.
- $\Phi_n = 0$ and $\Theta_n = 1$: the number of optical transmitters needed by a source is equal to that in the first case.
- $\Phi_n = 1$ and $\Theta_n = 0$: Since node *n* is capable of power splitting, each outgoing wavelength requires only one optical transmitter. *M* makes eq. (14) always hold for any assignment of values to the variables. Eqs. (15) and (16) ensure that the number of outgoing wavelengths of the source must be equal to or less than the number of optical transmitters in the source.
- $\Phi_n = 1$ and $\Theta_n = 1$: *M* makes eqs. (14)-(16) always hold. The number of light-trees rooted at node *n* must be equal to or less than the number of optical transmitters at node *n*. We assume that the given parameters always satisfy this constraint.

Eq. (17) states that the number of light-trees in which a node *n* participates must be equal to or less than the number of optical receivers at *n*. Since both $D_n^{(t)}$ and Rx_n are parameters, here we assume this constraint always holds. Later in the next section, we will need this constraint because $D_n^{(t)}$ becomes a variable.

B. Design of Light-Tree Based Logical Topologies

In this section, we extend the formulation described above to solve the logical topology design problem. A logical topology is a hypergraph, in which each node represents a switch capable of electronic processing. Each hyperedge represents a light-tree in the physical network. A node incapable of electronic processing can't be the root of a light-tree. We use the formulation of light-trees described in the last section to find the hyperedges for the logical topology. The traffic over the logical topology is composed of multiple *multicast streams* to which an end-to-end delay bound is guaranteed. Each multicast stream has a *sender* and several *receivers*. We assume that the propagation delay dominates the end-to-end delay because the link can be maintained relatively lightly utilized by setting $\lambda_{\max}$ properly, leading to negligible queueing delays. The goal of this section is to find the optimal design of logical topologies and the optimal routing of multicast streams using MILP. Some new parameters are defined as follows.

j	a multicast stream, $j \in H$;
λ_j	the data rate of multicast stream *j*;
$\lambda_{\max}$	the data rate supported on a wavelength channel;
$R_n^{(j)}$	if *n* is a receiver of stream *j*, $R_n^{(j)} = 1$; otherwise, $R_n^{(j)} = 0$;
$S_n^{(j)}$	if *n* is the sender of stream *j*, $S_n^{(j)} = 1$; otherwise, $S_n^{(j)} = 0$;
PS_j	the end-to-end delay bound of stream *j*;

Some new variables are defined as follows.

$B^{(t,j)}$	if the data of multicast stream *j* are carried on light-tree *t*, $B^{(t,j)} = 1$; otherwise, $B^{(t,j)} = 0$;
$Z_n^{(t,j)}$	if node *n* is a destination of light-tree *t*, and the data of multicast stream *j* are carried on light-tree *t*, $Z_n^{(t,j)} = 1$; otherwise, $Z_n^{(t,j)} = 0$;
$Y_n^{(j)}$	the end-to-end delay from the sender of stream *j* to node *n*, $Y_n^{(j)} \ge 0$;

$D_n^{(t)}$, as a parameter in the last section, becomes a variable in this section. Moreover, variable PT_t and the constraint in eq. (12) are of no use in this section. Here the value of *M* should be set as follows.

$$M = \max\left(\sum_{j\in H} PS_j, |E|\times|\Lambda|\times|\Gamma|\times|H|\times F_e, \sum_{e\in E} P_e\right) \qquad (18)$$

The objective function may either be to minimize the congestion [1,3] or to minimize the average packet hop distance [2]. For the former case, $\lambda_{\max}$ is set as a variable. For the latter case, additional variables representing the data rate of each stream carried on light-tree *t* transmitted on each link are required. In this paper, we use the same objective function as in the previous section (i.e., eq. (2)) in order to compare the following three design scenarios fairly.

- Each multicast stream is carried on a different light-tree. The problem is just the routing and wavelength assignment of light-trees as discussed in [7-9,12-14]. Optimal solution can be obtained using formulation in previous section.
- Multicasting is only available on logical networks. Each node of a logical network is capable of multicasting. Each edge of a logical network is a lightpath. It is the problem as discussed in [15].
- Multicasting is available both on physical networks and logical networks. A light-tree can carry data from multiple multicast streams. It is the focus of this section.

The set of constraints is listed as follows.

1) Multicast routing constraints:

$$R_n^{(j)} \times \sum_{t:t\in\Gamma, t\notin T_n} Z_n^{(t,j)} \geq R_n^{(j)}, \forall n \in V, \forall j \in H \quad (19)$$

$$B^{(t,j)} + D_n^{(t)} \geq 2 \times Z_n^{(t,j)}, \forall n \in V, \forall t \in \Gamma, t \notin T_n, \forall j \in H \quad (20)$$

$$\left(1 - S_n^{(j)}\right) \times \frac{1}{M} \times \sum_{t\in T_n} B^{(t,j)} \leq \left(1 - S_n^{(j)}\right) \times \sum_{t:t\in\Gamma, t\notin T_n} Z_n^{(t,j)}, \forall n \in V, \forall j \in H \quad (21)$$

Eqs. (19) to (21) ensure that data of each multicast stream will pass through one or several light-trees to each receiver. Eqs. (19) to (20) guarantee that each receiver of stream j must be a destination of a light-tree which carries the data of j. Eq. (21) ensures that the source of the light-tree which carries the data of stream j must be a destination of another light-tree which also carries the data of stream j, or the source must be the sender of stream j. Similar to the routing and wavelength assignment constraints in the last section, there must exist a path from the sender of j to each receiver, or the receiver must be in a loop. The delay constraints specified in the next paragraph will ensure that the latter case will not exist..

2) Delay constraints:

$$S_n^{(j)} \times Y_n^{(j)} = 0, \forall n \in V, \forall j \in H \quad (22)$$

$$Y_n^{(j)} \geq Y_m^{(j)} + Q_n^{(t)} - \left(1 - Z_n^{(t,j)}\right) \times M, \forall m, n \in V, m \neq n, \forall t \in T_m, \forall j \in H \quad (23)$$

$$R_n^{(j)} \times \left(PS_j - Y_n^{(j)}\right) \geq 0, \forall n \in V, \forall j \in H \quad (24)$$

Eqs. (22) to (24) ensure that the delay of the path from the sender of a multicast stream to each receiver must be equal to or less than the end-to-end delay bound of the stream. Eq. (22) states that the delay from the sender of stream j to a node n equals zero if n is the sender of stream j. Eq. (23) ensures that if node n is a destination of light-tree t rooted at m, and t carries the data of stream j, then the delay from the sender of stream j to n must be equal to or larger than the delay to node m plus the delay of the path from m to n. Eq. (24) guarantees that the delay from the sender of stream j to each receiver must satisfy the end-to-end delay bound.

3) Rate constraints:

$$\sum_{j\in H} \lambda_j \times B^{(t,j)} \leq \lambda_{\max}, \forall t \in \Gamma \quad (25)$$

Eq. (25) ensures that the data rate carried on each light-tree must be equal to or less than the maximum data rate which a wavelength channel can support

III. NUMERICAL EXAMPLES

In our experiments, the CPLEX optimization package [21] is used to obtain optimal solutions. We use a 14-node NSFNET backbone network as the topology of our numerical examples (as shown in Fig. 1). The number beside each node indicates the ID of the node. The number beside each link shows the propagation delay (in millisecond) of the link. We add some auxiliary constraints to improve the speed of finding optimal solutions. In our experiments, there is only one fiber on each link. The goal is to find the minimal number of wavelength channels in each experiment. Moreover, the minimal number of transmitters plus receivers is obtained by slightly modification of eq. (2)[2].

The first part of our experiments is to find the optimal routing and wavelength assignment of light-trees under different network parameters. The network parameters are listed as follows:

- the number of power splitters;
- the number of wavelength converters;
- the number of wavelengths supported in a fiber;
- the number of light-trees to be created;
- the number of destinations in a light-tree.

Fig. 2 shows the results of different number of power splitters when the number of destinations in a light-tree is changed. There are two light-trees rooted at nodes 2 and 11, respectively. The end-to-end delay bound of each light-tree is 100 ms. The destinations are chosen randomly. Each fiber supports five wavelengths. The network supports one wavelength converter. The results of more than one wavelength converters are similar. The results of light-trees with more destinations use more network resources. Besides, a small number of power splitters (e.g., 5 splitters) is enough to reduce the amount of resources used by the two light-trees. With more power splitters, the number of wavelength channels, optical transmitters, and receivers reduced by adding one more power splitter decreases. We note that a light-tree with more destinations benefits from more power splitters because more network nodes can be used as splitting nodes of the light-tree. With less power splitters, a separate lightpath has to be created from a further splitting node or the source to a destination.

Table I shows the placement of power splitters and wavelength converters in optimal solutions when each light-tree has 13 destinations. Table II focuses on the amount of resources used by the two light-trees with 11 and 13 destinations, respectively. Some experiments produce no feasible solutions. With more destinations, the network requires more power splitters, wavelength converters, or more number of wavelengths supported in a fiber to find feasible solutions. With more power splitters and wavelength converters, it is easier to obtain optimal solutions with less number of wavelengths supported in a fiber. The reason is that a light-tree in a network with more power splitters uses less number of wavelength channels. With more wavelength converters, more candidate wavelengths in a fiber can be selected by a light-tree.

[2] The total number of transmitters and receivers is added to eq. (2). However, its coefficient is much smaller than one to ensure that the optimal number of wavelength channels is not changed.

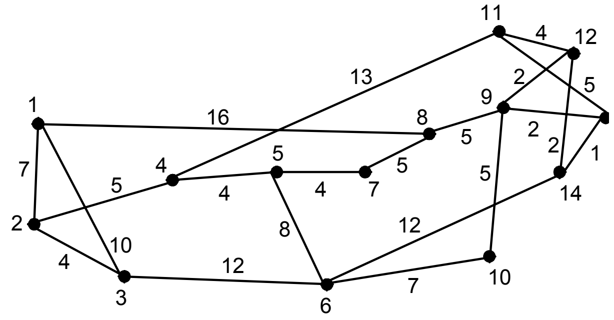

Figure 1. A 14-node NSFNET network

TABLE I
OPTIMAL PLACEMENT OF POWER SPLITTERS AND WAVELENGTH CONVERTERS

N_S	Splitter	Converter
1	6	6
2	6,9	6
3	6,8,9	9
4	4,5,6,9	6
5	2,4,5,9,13	4
6	4,5,6,8,9,10	4
7	2,4,8,9,10,11,13	4

Fig. 3 shows the results of different number of power splitters when the number of light-trees is changed. The source and destinations of each light-tree are chosen randomly. In average, there are six destinations in a light-tree. Each fiber supports four wavelengths. The network supports one wavelength converter. The results of more than one wavelength converters are similar. The results of more light-trees use more resources. The proportion of network resources reduced by power splitters is roughly the same, independent of the number of light-trees in a network. It is because the average numbers of destinations in all light-trees are identical.

The second part of our experiments is to compare the following strategies of transmitting two multicast streams in WDM networks.

- Two multicast streams are transmitted on two separate light-trees, without multiplexing together.
- Two multicast streams are multiplexed in logical networks.

We assume the data rate of a wavelength channel is larger than the sum of data rate required by the two streams. There are five nodes in the logical network, including the senders of the two streams. The senders, receivers, and the nodes in logical networks are chosen randomly. The end-to-end delay bound of each multicast stream is 100 ms. In average, there are six receivers in a multicast stream. There are three wavelengths in a fiber. The links of logical networks, i.e. RWA of lightpaths and light-trees, the placement of power splitters and wavelength converters, and the routing of the two multicast streams are to be decided in our experiments.

TABLE II
COMPARISONS OF RESOURCES USED BY TWO LIGHT-TREES WITH DIFFERENT NETWORK PARAMETERS

Number of wavelength channels										
D	$\|\Lambda\|$	N_C	N_S							
			0	1	2	3	4	5	6	7
11	1	0	X	X	X	X	X	X	26	24
11	1	1	X	X	X	X	X	X	26	24
11	2	0	X	X	33	29	26	24	23	23
11	2	1	X	X	33	29	25	24	23	23
11	3	0	X	40	33	29	26	24	23	23
11	3	1	X	36	32	28	25	24	23	23
11	4	0	48	39	33	29	26	24	23	23
11	4	1	48	38	32	28	25	24	23	23
13	1	0	X	X	X	X	X	X	X	26
13	1	1	X	X	X	X	X	X	X	26
13	2	0	X	X	X	34	30	28	26	26
13	2	1	X	X	40	32	29	27	26	26
13	3	0	X	51	40	34	30	28	26	26
13	3	1	X	43	37	32	29	27	26	26
13	4	0	X	47	40	34	30	28	26	26
13	4	1	X	42	37	32	29	27	26	26
13	5	0	59	50	40	34	30	28	26	26
13	5	1	59	42	37	32	29	27	26	26
Total number of transmitters and receivers										
D	$\|\Lambda\|$	N_C	N_S							
			0	1	2	3	4	5	6	7
11	1	0	X	X	X	X	X	X	27	26
11	1	1	X	X	X	X	X	X	27	26
11	2	0	X	X	34	31	28	25	24	24
11	2	1	X	X	33	30	27	25	24	24
11	3	0	X	37	34	31	28	25	24	24
11	3	1	X	33	30	30	27	25	24	24
11	4	0	44	36	34	31	28	25	24	24
11	4	1	44	33	30	28	27	25	24	24
13	1	0	X	X	X	X	X	X	X	30
13	1	1	X	X	X	X	X	X	X	30
13	2	0	X	X	X	38	34	30	28	28
13	2	1	X	X	38	35	32	29	28	28
13	3	0	X	44	38	37	34	30	28	28
13	3	1	X	40	35	35	31	29	28	28
13	4	0	X	44	38	37	34	30	28	28
13	4	1	X	37	35	33	29	29	28	28
13	5	0	52	38	38	37	34	30	28	28
13	5	1	52	37	35	33	29	29	28	28

D: the number of destinations in each light-tree
$|\Lambda|$: the number of wavelengths in a fiber
N_C : the number of wavelength converters in the network
N_S : the number of power splitters in the network
X: no feasible solution

Fig. 4 shows the results of different number of power splitters with different transmission strategies. The results are averaged over 10 samples. The results of multiplexing two streams in lightpath based logical networks are just the results of multiplexing in logical networks without power splitters, i.e., $N_S = 0$, the leftest point of the lower curve. Compared with lightpath based logical networks, multiplexing two

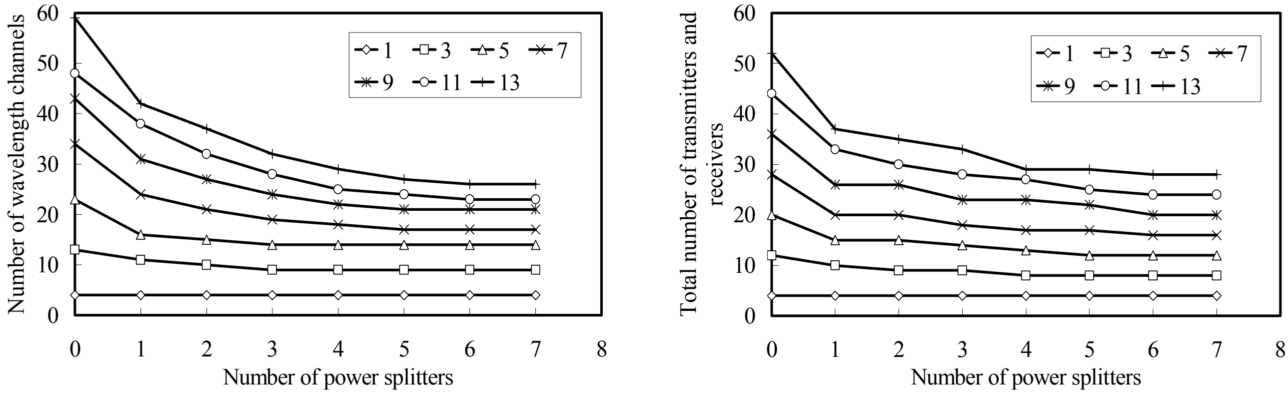

Figure 2. Comparisons of the resources used by two light-trees with different numbers of destinations in each light-tree.

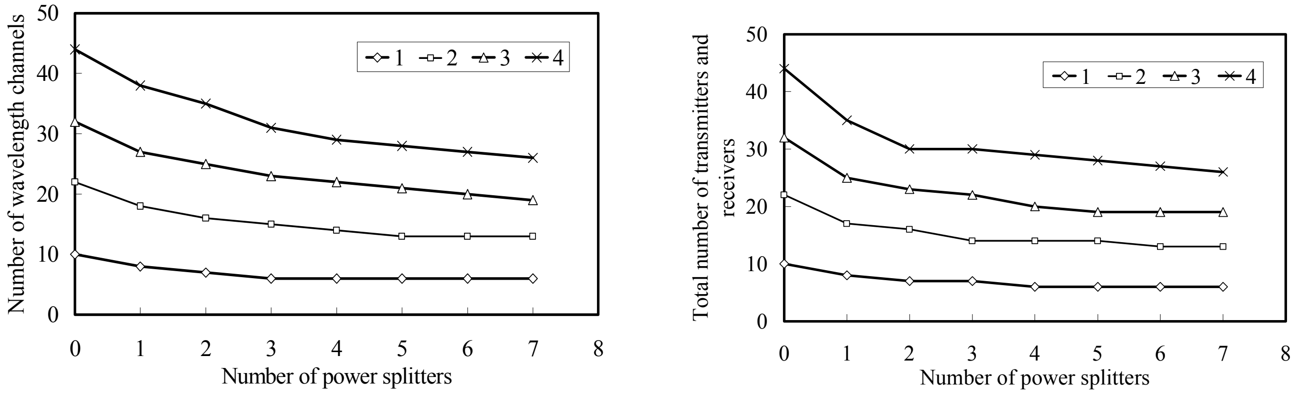

Figure 3. Comparisons of the resources used by different number of light-trees.

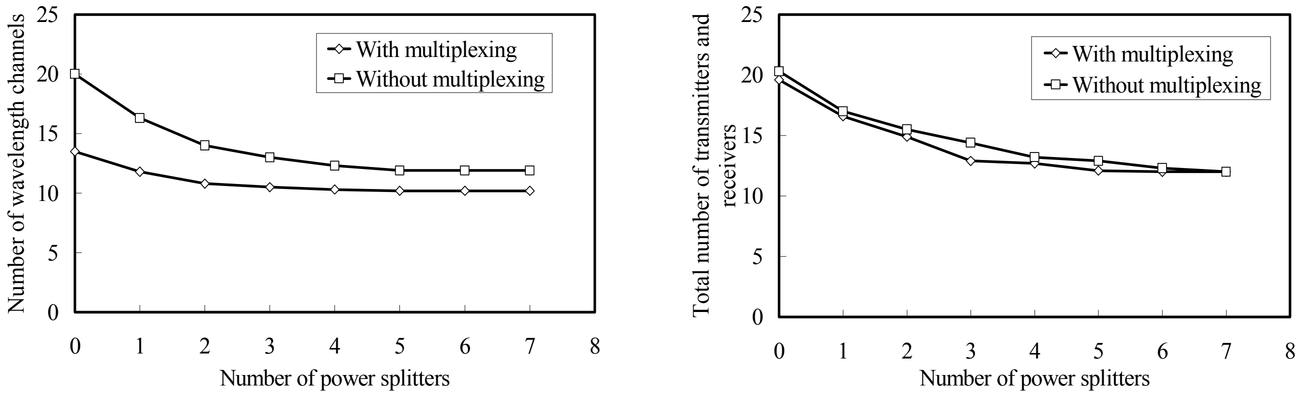

Figure 4. Comparisons of the resources used by two multicast streams with different transmission strategies.

streams in light-tree based logical network ,i.e., $N_S \geq 1$ uses less number of wavelength channels, optical transmitters, and receivers, with the help of wavelength converters and power splitters. In comparison with transmission on two separate light-trees, multiplexing in light-tree based logical networks uses less number of wavelength channels since both streams can be carried on a single wavelength channel. Increasing the number of power splitters reduces less number of wavelength channels when two streams are multiplexed in logical networks. The reason is that each node capable of electronic processing, i.e. the node in logical networks, can act as a splitting node of multicast tree in logical networks. The total numbers of transmitters and receivers required by transmission with and without multiplexing are close. The reasons are described as follows. For transmission on two separate light-trees, the source of a light-tree uses more transmitters. For multiplexing on light-tree based logical networks, even though the sender of a multicast stream can use fewer transmitters, nodes capable of electronic processing in a multicast tree need some transmitters and receivers to convert data between optical and electronic domains.

IV. Discussion

In this paper, we solve the routing, wavelength assignment of light-trees, placement of power splitters and wavelength converters, and the logical topology design by using MILP with a limited number of variables and constraints. The number of variables and constraints used in our formulation are $O(|\Gamma|\times(|E|\times|\Lambda|+|V|\times|H|))$ and $O(|V|^2\times|\Gamma|\times(|\Lambda|+|H|))$, respectively. Considering the routing and wavelength assignment light-trees or placement of power splitters, the number of variables and constraints in previous work [12,13] grows exponentially with network sizes in order to include all feasible light-trees in the formulation. The number of all possible light-trees in a network is then exponential with the network size. On the other hand, the number of variables and constraints in our formulation are only $O(|E|\times|\Lambda|\times|\Gamma|)$ and $O(|V|^2\times|\Gamma|\times|\Lambda|)$, respectively. Note that we do not compare with [14] because it does not consider the problem of wavelength assignment.

Considering the design of lightpath based logical topology for unicast traffic, the number of variables and constraints in [3] are both $O(q\times|V|^2\times(|E|\times|\Lambda|\times F_e+|H'|))$, where q is the maximal number of logical links which can be created for a source-destination pair, and H' is the number of unicast streams. For a fair comparison with our formulation, if q is the maximal number of light-trees which can be created for a source-destination pair, the number of light-trees in a network for our formulation should satisfy the following equation:

$$|\Gamma| = q\times|V| \tag{26}$$

According to eq. (26), our formulation needs much less number of variables. Note that we do not compare with [15] because it does not consider the problem of wavelength assignment.

Tornatore et al. [4] propose a novel formulation, named source formulation, which can reduce the number of variables and constraints. The idea is that for each link the set of variables representing the amount of flows from a source to each destination can be substituted by a single variable representing the total amount of flows from the source to all destinations. The flow conservation rule for the source formulation is that the amount of flows originating from a source incoming to a node is equal to the amount of flows outgoing from the node. Unfortunately, the source formulation cannot be applied here because the flow conservation rule does not hold for light-trees. Therefore, we cannot aggregate the set of variables corresponding to the light-trees with the same source into a single variable.

V. Conclusion

In this paper, we have presented a Mixed Integer Linear Programming (MILP) formulation to solve the optimal routing and wavelength assignment problem of light-trees with an end-to-end delay bound, and obtain the optimal placement of power splitters and wavelength converters in the network. The results show that networks with just a few power splitters and wavelength converters can efficiently carry multicast data. We have also extended the above formulation to design a light-tree based topology for multicast streams with an end-to-end delay bound, and obtain the optimal routing of multicast streams. We have demonstrated that this approach can use the network resources more efficiently, as compared to the approach in which each multicast stream is transmitted on a different light-tree, and the approach in which all streams are multicast in lightpath based logical networks.

Acknowledgment

This work was supported in part by MOE program for Promoting Academic Excellence of Universities under grant number 89E-FA06-2-4-7, and in part by the National Science Council, Taiwan, under grant number NSC91-2213-E-002-067.

References

[1] R. Ramaswami and K. N. Sivarajan, "Design of Logical Topologies for Wavelength-Routed Optical Networks," IEEE J. Select. Areas Commun., vol. 14, no. 5, Jun. 1996, pp. 840-851.

[2] D. Banerjee and B. Mukherjee, "Wavelength-Routed Optical Networks: Linear Formulation, Resource Budgeting Tradeoffs, and Reconfiguration Study," IEEE/ACM Trans. Networking, vol. 8, no. 5, Oct. 2000, pp. 198-607.

[3] R. M. Krishnaswamy, and K. N. Sivarajan, "Design of Logical Topologies: A Linear Formulation for Wavelength-Routed Optical Networks with No Wavelength Changers," IEEE/ACM Trans. Networking, vol. 9, no. 2, Apr. 2001, pp. 186-198.

[4] M. Tornatore, G. Maier, and A. Pattavina, "WDM Network Optimization by ILP Based on Source Formulation," IEEE INFOCOM'02.

[5] G. Sahin and M. Azizoglu, "Multicast Routing and Wavelength Assignment in Wide Area Networks", Proceedings of SPIE, vol. 3531, pp. 196-208, 1998.

[6] L. H. Sahasrabuddhe and B. Mukherjee, "Light-Trees: Optical Multicasting for Improved Performance in Wavelength-Routed Networks," IEEE Commun. Mag., Feb. 1999, pp. 67-73.

[7] X. Zhang, J. Wei, and C. Qiao," Constrained Multicast routing in WDM Networks with Sparse Light Splitting," IEEE INFOCOM'00, pp. 1781-1790.

[8] Xiao-Hua Jia et al., "Optimization of Wavelength Assignment for QoS Multicast in WDM Networks," IEEE Tran. Commun., vol. 49, no. 2, Feb. 2001, pp. 341-350.

[9] B. Chen and J. Wang, "Efficient Routing and Wavelength Assignment for Multicast in WDM Networks," IEEE J. Select. Areas Commun, vol. 20, no. 1, Jan. 2002, pp. 97-109.

[10] M. Gondran and M. Minoux, Graph and Algorithms, New York: Wiley, 1984.

[11] Noronha Jr. and F. A. Tobagi, "Optimum Routing of Multicast Streams," IEEE INFOCOM'94, pp. 865-873.

[12] M. Ali and J. S. Deogun., "Power-Efficient Design of Multicast Wavelength-Routed Networks," IEEE J. Select. Areas Commun, vol. 18, no. 10, Oct. 2000, pp. 1852-1862.

[13] M. Ali and J. Deogun "Allocation of Multicast Nodes in Wavelength-Routed Networks," IEEE ICC'01, vol. 2, pp.615-618.

[14] S. Yan , M. Ali, and J. Deogun, "Route Optimization of Multicast Sessions in Sparse Light-splitting Optical Networks," IEEE GLOBECOM'01, vol. 4, pp. 2134-2138.

[15] M. Mellia et al. "Optimal Design of Logical Topologies in Wavelength-Routed Optical Networks with Multicast Traffic," IEEE GLOBECOM'01, vol. 3, pp.1520-1525.

[16] S. Subramaniam et al., "All-Optical Networks with Sparse Wavelength Conversion," IEEE/ACM Trans. Networking, vol. 4, no. 4 Aug. 1996.

[17] G. Xiao and Y. Leung, “Algorithms for Allocating Wavelength Converters in All-Optical Networks,” IEEE/ACM Trans. Networking, vol. 7, no. 4, Aug. 1999, pp. 545-557.

[18] S. Subramaniam et al., “On Optimal Converter Placement in Wavelength-Routed Networks,” IEEE/ACM Trans. Networking, vol. 7, no. 5, Oct. 1999, pp. 754-766.

[19] S. Jiang, T. E. Stern, and E. Bouillet, “Design of Multicast Multilayered Lightwave Network,” IEEE GLOBECOM’93, vol. 1, pp. 452-457.

[20] S. Jiang and T. E. Stern, “Regular Multicast Multihop Lightwave Networks,” IEEE INFOCOM’95, vol. 2, pp. 692-700.

[21] ILOG CPLEX Optimization Package. http://www.ilog.com/products/cplex/