The Impact of Space Division Multiplexing on Resource Allocation: A Unified Approach

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Abstract— **Recent advances in the area of wireless communications have revealed the emerging need for efficient wireless access in personal, local and wide area networks. Space division multiple access (SDMA) with smart antennas at the base station is recognized as a promising means of increasing system capacity and supporting rate-demanding services. However, the existence of SDMA at the physical layer raises significant issues at higher layers. In this paper, we attempt to capture the impact of SDMA on channel allocation at the media access control (MAC) layer. This impact obtains different forms in TDMA, CDMA and OFDMA access schemes, due to the different cochannel and inter-channel interference instances, as well as the different effect of corresponding channels (time slots, codes or subcarrier frequencies) on user channel characteristics. We follow a unified approach for these multiple access schemes and propose heuristic algorithms to allocate channels to users and adjust down-link beamforming vectors and transmission powers, with the objective to increase achievable system rate and provide QoS to users in the form of minimum rate guarantees. We consider the class of greedy algorithms, based on criteria such as minimum induced or received interference and minimum signal-to-interference ratio (SIR), as well as the class of SIR balancing algorithms. Our results indicate that this cross-layer approach yields significant performance benefits and that SIR balancing algorithms achieves the best performance.**

I. Introduction

Recent evolutions in the telecommunications arena indicate a clear trend towards enhanced services that are expected to flourish in the near future. The advent of rate-demanding services such as tele-commuting, home networking, video conferencing, fast wireless/mobile internet access and multimedia constitutes only the first manifestation of the projected demand for broadband access to information sources of every kind. The need for ubiquitous coverage and connectivity in personal, local or wide area networks and the increasing demand for mobility, flexibility and easiness of system deployment necessitate wireless broadband access. Providing diverse quality of service (QoS) guarantees to users by using the scarce resources in the inherently volatile wireless medium is a challenging issue. At the physical layer, QoS is synonymous to an acceptable signal-to-interference-and-noise ratio (SINR) level or bit error rate (BER) at the receiver, while at the MAC or higher layers, QoS is usually expressed in terms of minimum rate or maximum delay guarantees. The fulfillment of QoS requirements depends on procedures that span several layers. At the MAC layer, QoS guarantees can be provided by appropriate scheduling [1] and channel allocation methods [2]. At the physical layer, adaptation of transmission power, modulation level or symbol rate helps in maintaining acceptable link quality [3], [4]. Moreover, smart antennas constitute perhaps the most promising means of increasing system capacity through space division multiple access (SDMA) [5].

Third generation (3G) TDMA-based IS-136+ is the successor of Enhanced Data for GSM Evolution (EDGE). Direct sequence CDMA (DS-CDMA) is also a major candidate for delivering the envisioned high and diverse data rates of proposed UMTS and cdma2000 3G systems. In DS-CDMA, user symbols are modulated by a high-rate chip sequence, the spreading code. The number of chips per symbol is called spreading gain. Multi-code structures and different spreading gains per code enable the support of high data rates for 3G systems [6]. Orthogonal frequency division multiple access (OFDMA) is another proposed access and signaling scheme for wireless broadband networks [7]. OFDMA is included in the IEEE 802.11a [8] and ETSI HIPERLAN/2 standards for wireless local area networks (WLANs). It has also been proposed by IEEE 802.15 and 802.16 working groups for wireless personal area networks (WPANs) and fixed broadband wireless access (FBWA). In OFDMA, the wide-band spectrum is divided into orthogonal narrow-band subcarriers as in frequency division multiplexing. The bit stream is split into subsets, each of which constitutes a subsymbol. Each subsymbol modulates a different subcarrier and several subsymbols of a user are transmitted in parallel over subcarriers. The orthogonality of signals in different subcarriers is preserved by appropriate spacing between subcarrier frequencies. OFDM transmission reduces the effective symbol transmission rate and thus provides high immunity to inter-symbol interference (ISI). The subcarrier spacing results in high spectral efficiency and high achievable data rates.

The early work in [9] considered power control to provide the maximum common SINR for a set of cochannel transmitter-receiver pairs. In [10], joint power control and rate adaptation for channel capacity maximization is studied. Existing literature on CDMA with deterministic codes focuses on code design so as to maximize channel capacity or minimize interference metrics, such as total square cross-correlation (TSC) of codes. In [11], code design and power allocation for sum capacity maximization are studied. The problem of assigning variable spreading gain deterministic codes to a set of users is addressed in [12]. For a single-cell multi-user OFDM system, the authors in [13] formulate the discrete subcarrier allocation problem as an integer programming one and find a suboptimal solution by using the continuous relaxation. A similar approach is followed in [14] for the dual problem of finding the optimal subcarrier allocation in order to minimize total transmitted power and satisfy a minimum rate constraint for each user. Channel allocation with modulation and power control in a multi-cell system is investigated in [15] for generic multiple access schemes with orthogonal channels.

Space division multiple access (SDMA) is recognized as the primary means of enhancing capacity in wireless networks. It can be combined with a multiple access scheme and it enables intra-cell channel reuse by several spatially separable users. The SDMA-based smart WLAN (SWL) system is proposed for cooperation with the 802.11 protocol [16]. Several companies such as Iospanwireless, Metawave, Navini and Arraycomm aim at SDMA commercial products that support certain multiple access schemes. Within each channel, multiple beams are formed by a smart antenna array at the base station. The radiation pattern of each beam is adjusted, so that the main lobe is steered to the direction of the desired user and nulls are placed in the directions of interfering users. Along with beamforming, power control is used for adjusting interference levels, so as to ensure acceptable SINRs at receivers. In the up-link, the user separation problem is decomposed into independent problems, one for each user, and beams are easily computed. However, user separation in the down-link is cumbersome, since beam adaptation for a user affects interference at all receivers. Furthermore, since receivers are distributed and are not expected to have multiple antennas, they cannot perform joint signal detection as in the up-link.

Down-link beamforming for power minimization in a single-cell system is studied in [17], where the problems of finding beams and powers are decoupled. In [18], the authors propose an iterative algorithm for joint transmit power control and receive beamforming for the up-link of a set of cochannel links. The algorithm converges to a feasible solution if there exists one and this solution minimizes total transmitted power over all feasible power allocations and beamforming vectors. A weak point of the approach is that the algorithm cannot detect infeasible solutions that lead to divergence if SINRs cannot be supported. The same authors in [19] solve the joint problem of power control and beamforming in the down-link by transforming it into an equivalent problem of transmit power control and receive beamforming for the up-link and applying the technique of [18]. In [20], [21], an iterative down-link beamforming and power control algorithm is presented, which always converges to the maximum common SIR for a set of cochannel links.

The employment of smart antennas at the physical layer raises significant issues at higher layers. In [22], [23], some heuristics for time slot assignment to users in an TDMA/SDMA system are proposed. In [24], the joint problem of subcarrier allocation, modulation control and beamforming in an OFDMA/SDMA system was studied. The problem was addressed for a system with or without channel reuse. For the former case, a methodology for constructing cochannel user sets with high total subcarrier rate was outlined. For the latter case, beamforming was considered as an additional dimension to enhance user SINR. Suboptimal heuristics for subcarrier allocation were proposed, with the goal to maximize total system rate.

With the exception of these works, channel allocation has hitherto been studied independently from user spatial separation through SDMA and from channel reuse. Intra-cell channel reuse is suboptimal and is usually based on static cell sectorization [25] or beam switching methods, which do not fully capture user mobility, channel dynamics and traffic load variations. Related research on beamforming has mostly focused on beam adaptation for a single channel in order to ensure acceptable SINR at each receiver. Thus, in a multi-channel system, beam adaptation for users is performed independently for each channel, without any consideration of its impact on users in other channels or on user QoS at the MAC layer. However, a particular allocation of users to channels affects the achievable system rate and the degree to which QoS is ensured for each user, because spatial channel characteristics of users may vary in different channels and because channels may be non-orthogonal. Therefore, it is important to identify appropriate cochannel sets of spatially separable users for each channel by taking into account cochannel and inter-channel interference.

In this paper, we investigate the impact of SDMA on MAC layer channel allocation, so as to increase system rate and provide minimum rate guarantees to users. We consider a generalized framework that encompasses OFDMA, TDMA and CDMA. In these multiple access schemes, different channels may have different effect on propagation characteristics of users. Different cochannel and inter-channel interference instances may also be created. We propose heuristic algorithms to assign users to channels, while appropriately adjusting beam directions and powers. We focus on the class of greedy assignment algorithms with criteria such as minimum induced or received interference and minimum user SIR, as well as on the class of SIR balancing algorithms. The main goals of our study are to evaluate the benefits of this cross-layer approach in terms of achievable rates, to identify the special features of the algorithms when employed in each multiple access scheme, and to motivate further research on cross-layer design.

The paper is organized as follows. In section II we present the transmission and channel models. In section III, we outline the rationale of our approach and describe the general framework and the proposed algorithms and in section IV we provide numerical results. Finally section V concludes our study. A few words about notation before we proceed. Vectors and matrices are shown with boldface letters. The cardinality of set $\mathcal{X}$ is $|\mathcal{X}|$. Superscripts $*$, T and H denote conjugate of a complex number, transpose and conjugate transpose of a vector or matrix and $\|\mathbf{u}\|=\sqrt{\sum_{i=1}^{n}|u_i|^2}$ is the ℓ_2-norm of complex vector $\mathbf{u}=(u_1,\ldots,u_n)^T$. The dominant generalized eigenvector of matrix pair $(\mathbf{A},\mathbf{B})$, $\mathbf{u}_{max}(\mathbf{A},\mathbf{B})$ is the normalized eigenvector that corresponds to the largest positive eigenvalue of eigen-problem $\mathbf{Ax}=\lambda\mathbf{Bx}$. When $\mathbf{A}$ and $\mathbf{B}$ are symmetric and positive definite, this is equivalent to eigen-problem $\mathbf{Cy}=\lambda\mathbf{y}$, with $\mathbf{C}=\mathbf{L}^{-1}\mathbf{A}(\mathbf{L}^{-1})^H$ and $\mathbf{y}=\mathbf{L}^H\mathbf{x}$, where $\mathbf{L}$ is a non-singular lower triangular matrix that appears in the Cholesky decomposition of $\mathbf{B}$, which is $\mathbf{B}=\mathbf{LL}^H$ [26].

II. SYSTEM MODEL

We consider the down-link of a single-cell system with K users and N channels. Depending on the multiple access scheme (TDMA, CDMA or OFDMA), channels are time slots, codes or subcarrier frequencies. The base station (BS) has a uniform linear array of M antennas, while each receiver has an omni-directional antenna. An underlying slotted transmission scheme

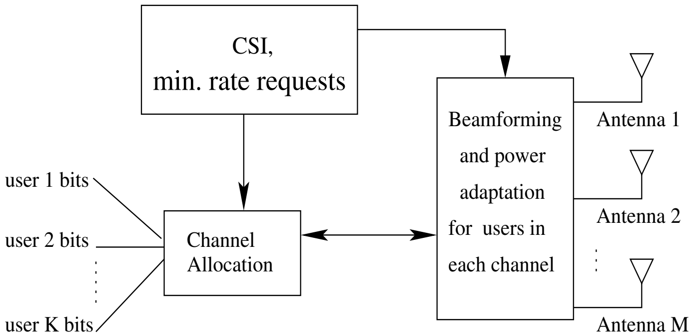

Fig. 1. SDMA transmitter with a generic multiple access scheme.

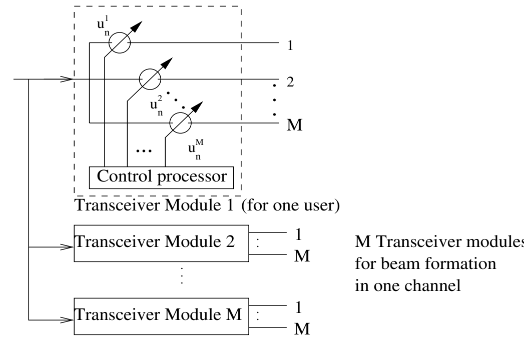

Fig. 2. The structure of M transceiver modules for channel n.

is assumed. Packetized data arrive from higher layers and are decomposed into bit streams before being transmitted to users. Channel quality remains constant for the duration of one time slot T_s, but may change between time slots. Each user k has a minimum rate requirement of r_k bits/sec over some time interval $(0, t)$, which consists of several slots. This requirement denotes QoS that the MAC layer requests from the physical layer. A fixed number of symbols S are transmitted in one slot and the symbol (signaling) interval is T. Single-rate transmission is assumed and r_k is mapped to a minimum number of required channels x_k.

The block diagram of a generic SDMA transmitter is depicted in figure 1. User bits enter the channel allocation module, which determines cochannel sets of users in different channels. Beamforming and power adaptation are subsequently performed for each user that is allocated to a channel. Under SDMA, the transmitter can form at most M beams for each channel and transmit to M cochannel users at the same time. A beam $\mathbf{u}_{n,k} = (u_{n,k}^1, \ldots, u_{n,k}^M)^T$ is formed by a dedicated transceiver and a power $p_{n,k}$ is assigned to user k in channel n. Beams are normalized to unit power, i.e, $\|\mathbf{u}_{n,k}\| = 1$. We assume that M transceivers (beamformers) are available for each channel, so that a separate beam can be formed for each one of the M users that can be separated in a channel. A set of M transceivers for a channel is shown in figure 2. Channel allocation and beamforming are interdependent operations and they also depend on user minimum rate requirements and channel state information (CSI), which are assumed to be available at the transmitter.

An OFDM/SDMA transmitter is shown in figure 3. After subcarrier allocation, beamforming and power adaptation, user bits are forwarded into M parallel modules of N modulators. Each modulator modulates the corresponding subcarrier with bits of users that are allocated to the subcarrier. For single-rate transmission, b bits of each assigned user modulate a subcarrier and constitute a subsymbol. Subsymbols of each user are then fed into an inverse discrete Fourier transform (IDFT) module and are transformed into N time domain samples, which form an OFDM user symbol. A cyclic prefix of some time samples is appended to eliminate ISI. After D/A conversion, continuous signals are transmitted from the M antennas. Assuming that OFDM symbols do not interfere with each other, it suffices to concentrate on one OFDM symbol. The transmitted base-band signal to user k from the mth antenna is

$$x_k^m(t) = \sum_{n=0}^{N-1} \sqrt{p_{n,k}}\, u_{n,k}^m d_{n,k} g(t) e^{j2\pi nt/T}, \quad 0 \le t \le T, \qquad (1)$$

where complex coefficient $d_{n,k}$ denotes the subsymbol of user k at the output of the nth modulator. If k uses n_k subcarriers, it achieves rate $(bS/T_s)n_k$ bits/sec in a time slot.

Multi-path characteristics for a user are similar across antennas, which accounts for relatively small spacing between antenna elements and no scatterers close to the BS, so that fading channels of different antennas are correlated. The multi-path channel between antenna m and user k is

$$h_k^m(t) = \sum_{\ell=1}^{L} \beta_{k,\ell}\, \delta\left(t - \tau_{k,\ell} + \tau_{k,\ell}^m\right), \qquad (2)$$

where L is the number of paths, $\beta_{k,\ell}$ is the complex gain of the ℓth path of user k and $\tau_{k,\ell}$ is its delay with respect to a reference antenna element. The term $\tau_{k,\ell}^m = (\delta/c)(m-1)\cos\theta_{k,\ell}$ captures delay difference between the mth antenna element and the reference element, where δ is the distance between two elements, $\theta_{k,\ell}$ is the angle of the ℓth path of user k with respect to the array and c is the electromagnetic wave propagation speed. The received signal for user k after down-conversion is

$$r_k(t) = \sum_{j=1}^{K} \sum_{m=1}^{M} \sum_{\ell=1}^{L} \beta_{k,\ell} x_j^m(t - \tau_{k,\ell} + \tau_{k,\ell}^m) + z(t), \qquad (3)$$

where $z(t)$ is the noise process. The received signal is digitized by being sampled at points (iT/N), $i = 0, \ldots, N-1$ and the N samples are fed into the DFT module. The subsymbol at subcarrier n is

$$y_{n,k} = \sum_{m=1}^{M} u_{n,k}^m \sum_{\ell=1}^{L} d_{n,k} \sqrt{p_{n,k}}\, \xi_{k,\ell}(n) e^{j2\pi(f_c + \frac{n}{T})\tau_{k,\ell}^m} + y_{n,k}^u + z_{n,k} \qquad (4)$$

where $y_{n,k}^u$ is the undesired interference from users other than k in subcarrier n, $z_{n,k}$ is the noise at subcarrier n, f_c is the carrier frequency and the factor

$$\xi_{k,\ell}(n) = \beta_{k,\ell} e^{-j2\pi(f_c + \frac{n}{T})\tau_{k,\ell}} \qquad (5)$$

captures the different impact of the ℓth path delay on different subcarriers of user k. Define the mth element of the $M \times 1$

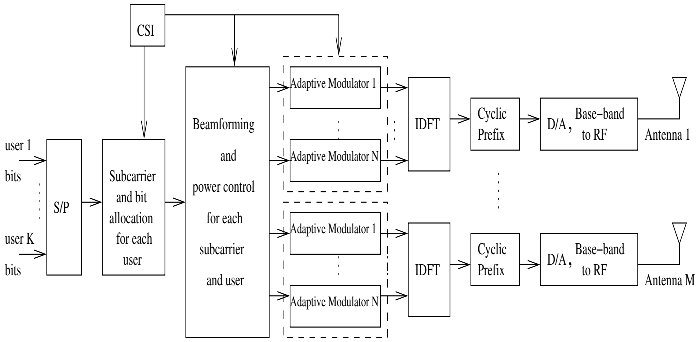

Fig. 3. Block diagram of a multi-user OFDM/SDMA transmitter.

antenna steering vector $\mathbf{v}_n(\theta_\ell)$ at subcarrier n for a path at direction θ_ℓ as $v_n^m(\theta_\ell) = e^{-j2\pi(f_c+n/T)\tau_\ell^m}$. Then, the vector

$$\mathbf{a}_{n,k} = \sum_{\ell=1}^{L} \xi_{k,\ell}^*(n)\mathbf{v}_n(\theta_{k,\ell}) \tag{6}$$

is called spatial signature of user k at subcarrier n and captures angular and multi-path properties of k at that frequency. The received signal for k at subcarrier n is

$$y_{n,k} = \sqrt{p_{n,k}}\left(\mathbf{a}_{n,k}^H \mathbf{u}_{n,k}\right) d_{n,k} + \sum_{\substack{j\in\mathcal{U}^{(n)}\\ j\neq k}} \sqrt{p_{n,j}}\left(\mathbf{a}_{n,k}^H \mathbf{u}_{n,j}\right) d_{n,j} + z_{n,k} \tag{7}$$

where $\mathcal{U}^{(n)}$ is the set of users in subcarrier n. If paths are resolvable, their angles, complex gains and delays are known to the transmitter and the SINR of user k at subcarrier n is

$$W_{n,k} = \frac{p_{n,k}\left(\mathbf{u}_{n,k}^H \mathcal{H}_{n,k}\mathbf{u}_{n,k}\right)}{\sum\limits_{\substack{j\in\mathcal{U}^{(n)}\\ j\neq k}} p_{n,j}\left(\mathbf{u}_{n,j}^H \mathcal{H}_{n,k}\mathbf{u}_{n,j}\right) + \sigma^2}\,, \tag{8}$$

where σ^2 is the noise variance and matrix $\mathcal{H}_{n,k}$ is defined as

$$\mathcal{H}_{n,k} = \sum_{\ell_1=1}^{L}\sum_{\ell_2=1}^{L} (\xi_{k,\ell_1}(n)\xi_{k,\ell_2}^*(n))\mathbf{v}_n(\theta_{k,\ell_1})\,\mathbf{v}_n^H(\theta_{k,\ell_2}). \tag{9}$$

In our model, we assume that cochannel interference is the prevailing interference type and that the noise level is not known. Then, the SINR is replaced by the signal-to-interference ratio (SIR). Apart from practical implications, this approach eliminates the need for total transmission power constraints.

If CSI is provided in terms of a statistical characterization of the parameters above, gains $\beta_{k,\ell}$ can be modeled as complex Gaussian random variables with zero mean and variance $A_{k,\ell}$ and delays $\tau_{k,\ell}$ are uniformly distributed in $[0,T]$. The expected useful received signal power is $\mathbb{E}\{|\sqrt{p_{n,k}}(\mathbf{a}_{n,k}^H\mathbf{u}_{n,k})d_{n,k}|^2\} = p_{n,k}(\mathbf{u}_{n,k}^H\mathcal{H}_{n,k}\mathbf{u}_{n,k})$, with

$$\mathcal{H}_{n,k} = \sum_{\ell_1=1}^{L}\sum_{\ell_2=1}^{L} \mathbf{v}_n(\theta_{k,\ell_1})\,\mathbf{v}_n^H(\theta_{k,\ell_2})\,\mathbb{E}\left\{\xi_{k,\ell_1}(n)\xi_{k,\ell_2}^*(n)\right\}, \tag{10}$$

where user symbols are normalized to unit power. If paths are uncorrelated,

$$\mathbb{E}\left\{\xi_{k,\ell_1}(n)\xi_{k,\ell_2}^*(n)\right\} = \begin{cases} 0, & \text{if } \ell_1 \neq \ell_2 \\ A_{k,\ell}, & \text{if } \ell_1 = \ell_2 = \ell \end{cases} \tag{11}$$

and

$$\mathcal{H}_{n,k} = \sum_{\ell=1}^{L} A_{k,\ell}\,\mathbf{v}_n(\theta_{k,\ell})\mathbf{v}_n^H(\theta_{k,\ell})\,. \tag{12}$$

Matrix $\mathcal{H}_{n,k}$ is called spatial covariance matrix of user k at subcarrier n and in general it has rank$(\mathcal{H}_{n,k}) > 1$, unless all paths have the same variance A_k, that is, they are identically distributed. The expected SINR is given by (8). Deterministic CSI at the transmitter is difficult to obtain in practice, since it requires knowledge of spatial signature, which means that angular and multi-path characteristics for each path are known. CSI in terms of spatial covariance matrices is more common. The spatial covariance matrix $\mathcal{H}_{n,k}$ can be estimated by sampling the received vector signal for each subcarrier in the up-link. Known pilot symbols can be used for this purpose. Then, the estimate of $\mathcal{H}_{n,k}$ is obtained by sample averaging. With reasonably small time variation of the channel, the BS can use this estimate to adapt the down-link beamforming vector.

In single-carrier TDMA, the N available channels are time slots. The transmitted base-band signal for user k that is assigned to slot n is $s_{n,k}(t) = \sum_i d_{n,k}(i)g(t-iT)$, where $\{d_{n,k}(i)\}$ is the symbol sequence, T is the symbol duration and $g(t)$ is the pulse shaping filter. If there is no pulse overlap, each symbol can be studied separately. By setting $i=0$, we have $s_{n,k}(t) = d_{n,k}g(t)$. At most M users can be assigned to a time slot. User signals are then multiplied by beamforming weights and powers and are transmitted from the M antennas. A modulation level of b bits/symbol is used and a rate of bS/T_s bits/sec is achieved for a user in a time slot. The received symbol for user k is

$$r_{n,k}(t) = \sum_{j\in\mathcal{U}^{(n)}} \sqrt{p_{n,j}} \sum_{m=1}^{M} u_{n,j}^m \sum_{\ell=1}^{L} \xi_{k,\ell}(\omega_c) e^{j2\pi f_c \tau_{k,\ell}^m} s_{n,j}(t), \tag{13}$$

where $\xi_{k,\ell}(\omega_c)$ is the complex gain of the ℓth path of user k at carrier frequency $\omega_c = 2\pi f_c$. For the received symbol of user k we obtain an expression similar to that in (7) and the SINR at the output of the matched filter of k at time slot n is given by (8). The difference of TDMA from OFDMA is that spatial signatures $\mathbf{a}_{n,k}$ and spatial covariance matrices $\mathcal{H}_{n,k}$ do not depend on frequency, but only on temporal channel variations between time slots. Furthermore, in OFDMA and TDMA $W_{n,k}$ does not depend on transmissions to users in other channels, due to channel orthogonality.

In CDMA, the N channels are deterministic real codes with fixed spreading gain G. Let $\mathbf{c}_n = (c_{n1},\ldots,c_{nG})$ denote the normalized vector corresponding to code n. Each code pair (n,m) is characterized by cross-correlation $\rho_{nm} = \mathbf{c}_n^T\mathbf{c}_m$, where $\rho_{nn} = 1$. In time domain, code n is represented as $c_n(t) = \sum_{r=1}^{G} c_{nr}p(t-(r-1)T_c)$, where $p(t)$ is the chip pulse and T_c is the chip duration. The transmitted signal to user k with code n is $s_{n,k}(t) = \sum_i d_{n,k}(i)c_n(t-iT)$. By setting $i=0$, a single symbol is denoted as $s_{n,k}(t) = d_{n,k}c_n(t)$. User signals are multiplied by beamforming weights and powers and are transmitted from the M antennas. Each code is associated with rate $1/(GT_c)$ and user k with n_k codes achieves rate $n_k/(GT_c)$ bits/sec.

The receiver of user k consists of a bank of matched filters, each of which is matched to a code used by the user. The signal at the output of the matched filter to code n is $y_{n,k} = \mathbf{c}_n^T \mathbf{y}_k$, where $\mathbf{y}_k$ is the signal at the input of the kth receiver. Note that $y_{n,k} = y_{n,k}^d + y_{n,k}^u$, where $y_{n,k}^d = \sqrt{p_{n,k}}(\mathbf{a}_k^H \mathbf{u}_{n,k}) d_{n,k}$ is the desired signal of user k that is transmitted with code n and $y_{n,k}^u$ is the undesired interference signal. The latter is

$$y_{n,k}^u = \sum_{\substack{j \in \mathcal{U}^{(n)} \\ j \neq k}} \sqrt{p_{n,j}}(\mathbf{a}_k^H \mathbf{u}_{n,j}) d_{n,j} + \sum_{\substack{m=1 \\ m \neq n}}^{N} \sum_{j \in \mathcal{U}^{(m)}} \rho_{nm} \sqrt{p_{m,j}}(\mathbf{a}_k^H \mathbf{u}_{m,j}) d_{m,j} \tag{14}$$

and the SIR at the output of the matched filter to code n of k is

$$W_{n,k} = \frac{p_{n,k}(\mathbf{u}_{n,k}^H \mathcal{H}_k \mathbf{u}_{n,k})}{\sum\limits_{\substack{j \in \mathcal{U}^{(n)} \\ j \neq k}} p_{n,j}(\mathbf{u}_{n,j}^H \mathcal{H}_k \mathbf{u}_{n,j}) + \sum\limits_{\substack{m=1 \\ m \neq n}}^{N} \sum\limits_{j \in \mathcal{U}^{(m)}} \rho_{nm}^2 p_{m,j}(\mathbf{u}_{m,j}^H \mathcal{H}_k \mathbf{u}_{m,j})} \tag{15}$$

where the first term in the denominator captures cochannel interference from other users that use code n and the second term denotes inter-channel interference from other utilized codes due to code cross-correlation. Note that spatial signatures $\mathbf{a}_k$ and spatial covariance matrices $\mathcal{H}_k$ are independent of channel n.

A single modulation level M_0 of b bits/symbol is employed. The minimum required SIR (in dB) in order to maintain BER $\leq \epsilon$ at the receiver is given by SIR threshold $\gamma = -(\ln(5\epsilon)/1.5)(M_0 - 1)$, as shown in [27].

As a final note, consider beamforming and power control in the up-link. The relationship between the up-link and the down-link will be used in the sequel. Beamforming is performed at BS and power is adapted for the up-link transmission of each user. The SIR for user k in channel n is

$$\tilde{W}_{n,k} = \frac{p_{n,k}\left(\mathbf{u}_{n,k}^H \mathcal{H}_{n,k} \mathbf{u}_{n,k}\right)}{\mathbf{u}_{n,k}^H \Big(\sum\limits_{\substack{j \in \mathcal{U}^{(n)} \\ j \neq k}} p_{n,j} \mathcal{H}_{n,j} \Big) \mathbf{u}_{n,k}} . \tag{16}$$

III. Resource Allocation in SDMA-based Networks: A Unified Approach

A. Problem Statement

SDMA allows intra-cell channel reuse by several users. Consider first the cases of TDMA and OFDMA with orthogonal channels. Two or more users are called *spatially separable* in a channel if they simultaneously receive transmitted useful signals in that channel and there exist beamforming vectors and powers for each user, such that minimum SIR requirements at corresponding receivers are satisfied, so that the specified BER can be maintained. For a given channel, spatial separability depends on the number and identities of individual users through their spatial covariance matrices, which in turn capture angular and multi-path characteristics of user channels. In addition, beamforming vectors and transmission powers affect interference levels and SIRs at all receivers and thus affect spatial separability. In single-carrier TDMA, user spatial separability is addressed for a time slot. The spatial covariance matrix of a user is the same for all slots of a time interval within which user channel characteristics do not change. In OFDMA, the frequency-selective broadband channel and the channel assignment in the frequency domain create an additional challenge. Spatial separability depends also on the individual subcarrier. Users that are separable in one subcarrier may not be separable in a different subcarrier. The dependence of spatial separability on subcarriers is attributed to the fact that different subcarrier frequencies have different impact on angular and multi-path characteristics of a user. In TDMA and OFDMA, a user in a channel experiences cochannel interference from beams of cochannel users. When many users are assigned to the same channel with SDMA, the total channel rate is increased. With higher channel reuse, users require fewer channels to satisfy rate requirements. Thus, more users can be accommodated in the system and capacity is increased. However, a large number of cochannel users renders spatial separability more difficult, since cochannel interference increases and user SIRs decrease.

In CDMA, multi-path characteristics of a user can be compensated at the receiver with the use of a RAKE path combiner after matched filtering and thus the impact of user channel characteristics on different codes is the same. Therefore, the spatial covariance matrix of each user is the same for all codes. The salient feature of CDMA is non-orthogonality of channels, due to pairwise code cross-correlations. A user that is assigned to a code receives cochannel interference from other users which use the same code, as well as inter-channel interference from other utilized correlated codes. Due to this interdependence among transmissions with different codes, user spatial separability must be defined with respect to all channels. A set of users is spatially separable with respect to a set of channels if there exist beamforming vectors and powers, one for each user in each utilized channel, such that minimum SIR requirements are satisfied for each user. A large number of users in a channel increases channel rate and contributes to system capacity increase, but it also has negative effect on spatial separability in that channel and other correlated channels.

The tradeoff between cardinality of cochannel user set and user spatial separability affects achievable system rate. The arising issue is whether there exists a way to perform channel allocation and user spatial separation jointly, so as to increase system rate and provide QoS guarantees to users. For TDMA and OFDMA, a large number of spatially separable cochannel users should be assigned to each channel. This is possible if users are spatially well separated. For users with a LOS path, spatial separability is easier if they are well separated in angle. For the more general case, spatial signatures of users should not be highly correlated and spatial covariance matrices and beamforming vectors should be such that users do not induce much interference to each other. The identification of the spatially separable cochannel user set with maximum channel rate is a hard optimization problem. First, a set of (at most M) spatially separable users must be identified. Then, beamforming vectors and powers must be computed for these users, so that minimum SIRs are satisfied. The problem is that the SIR at a receiver depends on beamforming vectors and powers of all users. The enumeration of all possible user assignments in a channel is of exponential complexity.

In addition, even if the spatially separable cochannel user set is given, the computation of beamforming vectors and powers that maximize channel rate is a highly non-linear problem. For CDMA, cochannel user sets in different channels create inter-channel interference among themselves. Users should be assigned to codes, so that user spatial characteristics, code cross-correlations, beamforming vectors and powers create a spatially separable cochannel user sets in all channels.

The discussion above necessitates the adoption of heuristic algorithms for constructing spatially separable cochannel user sets with appropriate beamforming and power control. In the sequel, we study three such algorithms. The first two fall within the category of greedy heuristics, but utilize different criteria for assignment of users to channels, namely minimum induced or received interference to or from other users and minimum user SIR. The third algorithm follows a different approach in channel allocation and tries to maintain the highest possible common SIR in each channel, by jointly adapting beamforming vectors and powers. The algorithms are described for the general case that encompasses TDMA, OFDMA and CDMA. Namely, we consider non-orthogonal channels and different spatial covariance matrices for a user in different channels.

B. Algorithm A

1) Beamforming vector adaptation: The basic idea is to form large cochannel sets of spatially separable users in each channel. In order to keep complexity to a reasonable level, we consider algorithms for which users are sequentially inserted in the channel and no channel reassignments are performed. Beamforming adjustment is allowed and power control is considered only when beamforming alone is insufficient for ensuring required user SIRs. At each step of the algorithm, an appropriate user is assigned to a channel and beamforming vectors of other users are adjusted, so that acceptable SIRs are ensured. An inserted user in a channel should induce the least cochannel and inter-channel interference to users that are already assigned in that channel and in other channels. It should also receive small amount of cochannel and inter-channel interference from other users.

Fix attention to channel n and let k be the user to be inserted next in n. Let $\mathbf{u}_{m,j}$ and $p_{m,j}$ denote the beamforming vector and power of user j in channel m. Insertion of user k in n creates a new interference instance for cochannel users in channel n and for users in other correlated channels m. Thus beamforming vectors that result in acceptable SIRs are needed. For each user $j \in \mathcal{U}^{(m)}$ define the ratio of desired power generated by beam $\mathbf{u}_{m,j}$, over interference power which is caused by $\mathbf{u}_{m,j}$ to other users, including the new user k in channel n. In fact, we are interested in the maximum value of this ratio, $\Psi_{m,j}^{(n,k)}$ over all directions $\mathbf{u}_{m,j}$,

$$\Psi_{m,j}^{(n,k)} = \max_{\mathbf{u}_{m,j}} \frac{\mathbf{u}_{m,j}^H \mathcal{H}_{m,j} \mathbf{u}_{m,j}}{\mathbf{u}_{m,j}^H \Bigg(\sum_{\substack{\mu=1 \\ \mu \neq m}}^{N} \sum_{i \in \mathcal{U}^{(\mu)}} \rho_{\mu m}^2 \mathcal{H}_{\mu,i} + \sum_{\substack{i \in \mathcal{U}^{(m)} \\ i \neq j}} \mathcal{H}_{m,i} + \rho_{mn}^2 \mathcal{H}_{n,k} \Bigg) \mathbf{u}_{m,j}} \tag{17}$$

such that $\|\mathbf{u}_{m,j}\| = 1$. Vector $\mathbf{u}_{m,j}^*$ that maximizes this ratio is the dominant generalized eigenvector of the pair of matrices that appear in the numerator and denominator and it is computed with the method outlined at the end of section I. We also compute the corresponding ratio for user k that is tentatively inserted in channel n,

$$\Psi_{n,k} = \max_{\mathbf{u}_{n,k}} \frac{\mathbf{u}_{n,k}^H \mathcal{H}_{n,k} \mathbf{u}_{n,k}}{\mathbf{u}_{n,k}^H \Bigg(\sum_{j \in \mathcal{U}^{(n)}} \mathcal{H}_{n,j} + \sum_{\substack{m=1 \\ m \neq n}}^{N} \sum_{i \in \mathcal{U}^{(m)}} \rho_{mn}^2 \mathcal{H}_{m,i} \Bigg) \mathbf{u}_{n,k}}, \tag{18}$$

such that $\|\mathbf{u}_{n,k}\| = 1$, where the denominator captures cochannel and inter-channel interference caused by the beam of k to other users. With the computed beamforming vectors, we evaluate the SIRs for user k in channel n and for users in other channels.

2) Power adaptation: If SIRs of some users do not exceed the minimum SIR γ, we fix the computed beamforming vectors and activate power control. Given a set of assigned users to some channels, the question is whether there exist powers, so that all SIRs exceed γ. For each channel i, let κ_i, ℓ_i be indices of users in that channel. Define $\mathbf{U}$ as the ensemble of computed beamforming vectors for users and channels, i.e, $\mathbf{U} = \{\mathbf{u}_{n,k} : k \in \mathcal{U}^{(n)},\, n = 1, \ldots, N\}$. Then, we define the $(\sum_{n=1}^N |\mathcal{U}^{(n)}|) \times (\sum_{n=1}^N |\mathcal{U}^{(n)}|)$ block matrix $\mathbf{A}(\mathbf{U})$ as

$$\mathbf{A}(\mathbf{U}) = \begin{pmatrix} \mathbf{A}_{11}(\mathbf{U}) & \mathbf{A}_{12}(\mathbf{U}) & \ldots & \mathbf{A}_{1N}(\mathbf{U}) \\ \mathbf{A}_{21}(\mathbf{U}) & \mathbf{A}_{22}(\mathbf{U}) & \ldots & \mathbf{A}_{2N}(\mathbf{U}) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{N1}(\mathbf{U}) & \mathbf{A}_{N2}(\mathbf{U}) & \ldots & \mathbf{A}_{NN}(\mathbf{U}) \end{pmatrix} \tag{19}$$

The $[\kappa_i, \ell_i]$th element of the $(|\mathcal{U}^{(i)}| \times |\mathcal{U}^{(i)}|)$ matrix $\mathbf{A}_{ii}(\mathbf{U})$ in the diagonal of $\mathbf{A}(\mathbf{U})$ specifies interference that is caused by the beam of the ℓ_ith user to the receiver of the κ_ith user in channel i, namely

$$\mathbf{A}_{ii}(\mathbf{U})[\kappa_i, \ell_i] = \begin{cases} \mathbf{u}_{i,\ell_i}^{*H} \mathcal{H}_{i,\kappa_i} \mathbf{u}_{i,\ell_i}^* & \text{if } \kappa_i \neq \ell_i \\ 1, & \text{if } \kappa_i = \ell_i. \end{cases} \tag{20}$$

The $[\kappa_i, \ell_j]$th element of the $(|\mathcal{U}^{(i)}| \times |\mathcal{U}^{(j)}|)$ matrix $\mathbf{A}_{ij}(\mathbf{U})$, $i \neq j$, denotes the inter-channel interference caused by the beam of the ℓ_jth user in channel j to the receiver of the κ_ith user in channel i,

$$\mathbf{A}_{ij}(\mathbf{U})[\kappa_i, \ell_i] = \rho_{ij}^2\, (\mathbf{u}_{j,\ell_j}^{*H} \mathcal{H}_{i,\kappa_i} \mathbf{u}_{j,\ell_j}^*). \tag{21}$$

Define also the diagonal matrix $\mathbf{\Delta}$,

$$\mathbf{\Delta} = \mathrm{diag}\Big\{ \frac{1}{\mathbf{u}_{i,\kappa_i}^{*H} \mathcal{H}_{i,\kappa_i} \mathbf{u}_{i,\kappa_i}^*},\, i = 1, \ldots, n,\, \kappa_i = 1, \ldots, |\mathcal{U}^{(i)}| \Big\} \tag{22}$$

and the $(\sum_{n=1}^N |\mathcal{U}^{n)}|) \times 1$ vector $\mathbf{p}$ of transmission powers to users in all channels. Then, the requirement $W_{n,k} \geq \gamma$ can be written in matrix form as

$$\frac{1+\gamma}{\gamma} \mathbf{p} \geq \mathbf{\Delta} \mathbf{A}(\mathbf{U}) \mathbf{p}. \tag{23}$$

Matrix $\mathbf{\Delta}\mathbf{A}(\mathbf{U})$ is non-negative definite and irreducible. From the Perron-Frobenius theorem, it has a positive, real eigenvalue $\lambda_{max}(\mathbf{\Delta}\mathbf{A}(\mathbf{U})) = \max_i \{|\lambda_i|\}$, where λ_i, $i = 1, \ldots, (\sum_n |\mathcal{U}^{(n)}|)$ are the eigenvalues of $\mathbf{\Delta}\mathbf{A}(\mathbf{U})$. The eigenvalue $\lambda_{max}(\mathbf{\Delta}\mathbf{A}(\mathbf{U}))$ has an associated eigenvector with strictly positive entries. Furthermore, the minimum real λ for which the inequality $\lambda \mathbf{p} \geq \mathbf{\Delta A}(\mathbf{U})\mathbf{p}$ has solutions $\mathbf{p} > 0$ is $\lambda = \lambda_{max}(\mathbf{\Delta A}(\mathbf{U}))$. In our case, we start by finding the maximum real positive eigenvalue of $\mathbf{\Delta A}(\mathbf{U})$ to request the existence of a power vector with positive entries. If $\lambda_{max}(\mathbf{\Delta A U}) \leq (1+\gamma)/\gamma$, then (23) holds and SIR level γ is achievable. The power vector that leads to an achievable γ is the eigenvector that corresponds to $\lambda_{max}(\mathbf{\Delta A}(\mathbf{U}))$.

Next, we define an assignment preference factor $\Phi_{n,k}$ for channel n and user k. First, the beam and power must yield strong desired signal for user k. Furthermore, all beams and powers should be such that interference $I_{n,k}$ caused by user k to other users, as well as induced interference $I'_{n,k}$ on k by other users is low. These requirements are captured by ratio

$$\Phi_{n,k} = \frac{p_{n,k}(\mathbf{u}_{n,k}^{*H}\mathcal{H}_{n,k}\mathbf{u}_{n,k}^{*})}{\max\left\{I_{n,k}\,,\, I'_{n,k}\right\}} \tag{24}$$

where $I_{n,k}$ and $I'_{n,k}$ are

$$I_{n,k} = p_{n,k}\mathbf{u}_{n,k}^{*H}\Big(\sum_{j\in\mathcal{U}^{(n)}}\mathcal{H}_{n,j} + \sum_{\substack{m=1\\ m\neq n}}^{N}\sum_{i\in\mathcal{U}^{(m)}}\rho_{mn}^{2}\mathcal{H}_{m,i}\Big)\mathbf{u}_{n,k}^{*} \tag{25}$$

$$I'_{n,k} = \sum_{j\in\mathcal{U}^{(n)}} p_{n,j}(\mathbf{u}_{n,j}^{*H}\mathcal{H}_{n,k}\mathbf{u}_{n,j}^{*}) + \sum_{\substack{m=1\\ m\neq n}}^{N}\sum_{i\in\mathcal{U}^{(m)}}\rho_{mn}^{2}p_{m,i}(\mathbf{u}_{m,i}^{*H}\mathcal{H}_{n,k}\mathbf{u}_{m,i}^{*}) \tag{26}$$

Clearly, if power control is not activated (when all SIRs exceed γ after initial beam computations with (17) and (18)), the ratios above do not include powers.

At each step of the algorithm, $\Phi_{n,k}$'s are computed for all channels n, for which a user insertion leads to acceptable SIRs and for all users k that have not satisfied minimum rate requirements x_k. Among assignments that yield acceptable SIRs for users, we select the one with the maximum $\Phi_{n,k}$. After each assignment, the rate of user k is updated. When a user reaches x_k, it is not considered for assignment until all users reach their minimum rate requirements. Then, all users are again considered for assignment. If the cardinality of a cochannel user set reaches M, the corresponding channel is not included for user assignment. The algorithm terminates when no further assignments are possible to any channel.

C. Algorithm B

The second class of algorithms is based on the criterion of maximizing the minimum SIR of users. In Algorithm A, a user that causes and receives the least interference is preferable for assignment. By following this greedy approach of least incremental interference, we aim at inserting as many users as possible in channels. In algorithm B, a user assignment in a channel is performed if it maximizes the minimum SIR of users in the system over all possible user assignments. Algorithm B does not simply consider induced and received interference, but it also attempts to capture the impact of an assignment on other users, so that SIRs that are close to the SIR threshold are maximized. With this assignment, we intend to facilitate future assignments and ultimately increase the number of users with SIRs above γ. The assignment factors are now defined as

$$\Phi_{n,k} = \min\big\{W_{n,k},\ \min_{\substack{j\in\mathcal{U}^{(m)}\\ m=1,\ldots,N}} W_{m,j}\big\}. \tag{27}$$

D. Algorithm C

Algorithms A and B perform sequential assignment of users in channels based on different greedy criteria. Recall that beamforming and power control were decoupled, since fixed beams were used to find feasible powers. Algorithm C follows a different approach, in the sense that it attempts to provide the maximum common SIR for users in the system. A salient feature of algorithm C is that it performs joint adaptation of beamforming vectors and powers in order to obtain the highest common SIR.

1) Single-channel algorithm: Consider channel n in isolation with the set of users $\mathcal{U}^{(n)}$. Let $\mathbf{p}_n$ and $\mathbf{U}_n$ be the power vector and the ensemble of computed beamforming vectors for users in n. Define the $(|\mathcal{U}^{(n)}| \times |\mathcal{U}^{(n)}|)$ matrix $\mathbf{B}(\mathbf{U}_n)$ with elements

$$\mathbf{B}(\mathbf{U}_n)[i,j] = \begin{cases} \mathbf{u}_{n,j}^{H}\mathcal{H}_{n,i}\mathbf{u}_{n,j} & \text{if } i\neq j \\ 0, & \text{if } i=j. \end{cases} \tag{28}$$

$\mathbf{B}(\mathbf{U}_n)$ is the interference matrix for users in channel n. Define also the diagonal matrix

$$\mathbf{D} = \text{diag}\big\{\frac{1}{\mathbf{u}_{n,i}^{H}\mathcal{H}_{n,i}\mathbf{u}_{n,i}} : i\in\mathcal{U}^{(n)}\big\}. \tag{29}$$

An instance in which all users achieve a common SIR γ_c in the *down-link* by using the ensemble of beamforming vectors $\mathbf{U}_n$ and power vector $\mathbf{p}_n$ is described by the set of equations

$$\mathbf{DB}(\mathbf{U}_n)\mathbf{p}_n = \frac{1}{\gamma_c}\mathbf{p}_n\,. \tag{30}$$

Thus, γ_c is a reciprocal eigenvalue of matrix $\mathbf{DB}(\mathbf{U}_n)$. Matrix $\mathbf{DB}(\mathbf{U}_n)$ has the same properties as $\mathbf{\Delta A}(\mathbf{U})$ with respect to the existence of an eigenvector $\mathbf{p}_n$ with positive entries. Therefore, we have $1/\gamma_c = \lambda_{max}(\mathbf{DB}(\mathbf{U}_n))$. Therefore, the maximum possible common SIR γ_c^* is

$$\gamma_c^* = \frac{1}{\min\limits_{\mathbf{U}_n}\lambda_{max}(\mathbf{DB}(\mathbf{U}_n))}\,. \tag{31}$$

We now consider the corresponding problem of beamforming and power control for the same users in the *up-link*. It can be verified that the instance in which all users achieve a common SIR $\tilde{\gamma}_c$ in the up-link by using an ensemble of beamforming vectors $\tilde{\mathbf{U}}_n$ and transmit power vector $\tilde{\mathbf{p}}_n$ is described by the set of equations,

$$\mathbf{DB}^{T}(\tilde{\mathbf{U}}_n)\tilde{\mathbf{p}}_n = \frac{1}{\tilde{\gamma}_c}\tilde{\mathbf{p}}_n \tag{32}$$

and the maximum possible common SIR $\tilde{\gamma}_c^*$ for the up-link is

$$\tilde{\gamma}_c^* = \frac{1}{\min\limits_{\tilde{\mathbf{U}}_n}\lambda_{max}(\mathbf{DB}^{T}(\tilde{\mathbf{U}}_n))}\,. \tag{33}$$

For the relationship between the down-link problem (31) and the up-link problem (33), the following properties have been proved in [20], [21]:

Property 1: For a given set of beamforming vectors $\mathbf{U}_n$, it is $\lambda_{max}(\mathbf{DB}(\mathbf{U}_n)) = \lambda_{max}(\mathbf{DB}^T(\mathbf{U}_n))$.

Property 2: The up-link and down-link problems have the same solution in terms of maximum achievable common SIR, i.e., it is $\gamma_c^* = \tilde{\gamma}_c^*$.

Property 3: The beamforming vectors that solve the down-link problem (31) and the up-link problem (33) are the same, namely $\mathbf{U}_n^* = \tilde{\mathbf{U}}_n^*$.

Property 4: In the following iterative algorithm (algorithm I), the sequence of eigenvalues $\lambda_{max}^{(t)}$ is monotonically decreasing with the iteration number t and the algorithm converges to a minimum, which is related with the maximum common SIR through (31) and (33).

ALGORITHM I

- **STEP 1**: Set $t = 0$. Start with arbitrary beamforming vectors $\mathbf{U}_n^{(0)}$.
- **STEP 2**: $t \leftarrow t+1$. For given $\mathbf{U}_n^{(t)}$, solve the following eigen-problem for the *uplink*:

$$\mathbf{DB}^T(\mathbf{U}_n^{(t)})\mathbf{p}_n^{(t)} = \lambda_{max}^{(t)}\mathbf{p}_n^{(t)} \,. \tag{34}$$

- **STEP 3**: For the computed $\mathbf{p}_n^{(t)}$, solve a set of *decoupled* generalized eigen-problems

$$\mathbf{u}_{n,k} = \arg\max_{\mathbf{u}_{n,k}} \frac{\mathbf{u}_{n,k}^H \mathcal{H}_{n,k}\mathbf{u}_{n,k}}{\mathbf{u}_{n,k}^H \mathcal{R}_{n,k}(\mathbf{p}_n^{(t)})\mathbf{u}_{n,k}^H}, \tag{35}$$

subject to $\|\mathbf{u}_{n,k}\| = 1$, for all $k \in \mathcal{U}^{(n)}$, where

$$\mathcal{R}_{n,k}(\mathbf{p}_n^{(t)}) = \sum_{\substack{j\in\mathcal{U}^{(n)}\\ j\neq k}} p_{n,j}^{(t)}\mathcal{H}_{n,j} \,. \tag{36}$$

- **STEP 4**: With the computed $\mathbf{U}_n^{(t)}$, go to step 2. Continue until convergence.

In step 3, the quantity to be maximized is the up-link SIR of user k. The beamforming vectors $\mathbf{U}_n^*$ at the end of the algorithm are the required down-link beams. If $\lambda_{max}^* = \lambda_{max}(\mathbf{DB}^T(\mathbf{U}_n^*))$ is the eigenvalue at the end of the algorithm, the down-link power vector is given by the eigenvector of $\mathbf{B}(\mathbf{U}_n^*)$ that corresponds to λ_{max}^*.

2) Description of Algorithm C: The rationale of Algorithm C is based on a simple observation that eliminates the need for channel assignment in a multi-user multi-channel system, where channel allocation, beamforming and power adaptation need to be performed jointly. A K-user system with N non-orthogonal channels can be viewed as a single-channel system. For a given user assignment to channels, users interfere with each other based on their spatial covariance matrices and channel cross-correlations. For this system we define the block interference matrix $\mathbf{F}(\mathbf{U})$ with matrix elements

$$\mathbf{F}_{ij}(\mathbf{U}) = \begin{cases} \mathbf{A}_{ij}(\mathbf{U}), & \text{if } i \neq j \\ \mathbf{A}_{ii}(\mathbf{U}) - \mathbf{I}, & \text{if } i = j, \end{cases} \tag{37}$$

where $\mathbf{I}$ is the identity matrix and the matrix elements of block matrix $\mathbf{A}(\mathbf{U})$ are defined in (20),(21). A system in which all users achieve a common SIR γ_c in the down-link is described by the set of linear equations $\mathbf{\Delta F}(\mathbf{U})\mathbf{p} = (1/\gamma_c)\mathbf{p}$. The correspondence with the single-channel system in (30) is obvious. In step 3 of algorithm I, a set of decoupled generalized eigen-problems are solved, for which

$$\mathcal{R}_{n,k}(\mathbf{p}^{(t)}) = \sum_{\substack{j\in\mathcal{U}^{(n)}\\ j\neq k}} p_{n,j}^{(t)}\mathcal{H}_{n,j} + \sum_{\substack{m=1\\ m\neq n}}^{N}\sum_{j\in\mathcal{U}^{(m)}} \rho_{nm}^2 p_{m,j}^{(t)}\mathcal{H}_{m,j}. \tag{38}$$

For a given assignment of users to channels, let γ_c^* denote the maximum common SIR which is computed by applying algorithm I. For each user $k \in \mathcal{U}^{(n)}$, let $\gamma_{c,n}(k)$ denote the common SIR of remaining users when k is removed from subcarrier n. Again, $\gamma_{c,n}(k)$ is found by Algorithm I, after deleting the appropriate row and column from $\mathbf{F}^T(\mathbf{U})$. The main steps of algorithm C are as follows:

- **STEP 0** : Start by assigning all K users in each channel n.
- **STEP 1** : Run algorithm I and find maximum common SIR γ_c^* of users.
- **STEP 2** : If $\gamma_c^* \geq \gamma$, desirable SIRs are achieved for all users and algorithm is terminated. Otherwise, go to step 3.
- **STEP 3** : For each $k \in \mathcal{U}^{(n)}$, compute $\gamma_{c,n}(k)$.
- **STEP 4** : Select pair (n^*, k^*) with maximum $\gamma_{c,n}(k)$ and remove user k^* from channel n^*.
- **STEP 5** : Update user rates. If a user reaches minimum rate requirements, do not consider it for further removal. Go to step 2.

By removing the user that yields the highest common SIR each time, we intend to remove fewer users until a desired common SIR is obtained, so as to achieve high system rate.

E. TDMA, OFDMA and CDMA as special cases

The three algorithms were presented for the general case that encompasses TDMA, OFDMA and CDMA. Algorithms A and B are applicable to these multiple access schemes with minor modifications, that reflect their special features. In single-carrier TDMA with orthogonal time slots, we have $\rho_{nm} = 0$ for different slots n, m. In a time interval of N slots where channel quality does not change for user k, the spatial covariance matrix of k is the same across all channels, i.e., $\mathcal{H}_{n,k} = \mathcal{H}_k$, for $n = 1, \ldots, N$. The spatial covariance matrix varies according to temporal channel variations from slot to slot. The interference factors given by (17) and (18) are now defined as $\Psi_j^{(n,k)}$ only for cochannel users $j \in \mathcal{U}^{(n)}$ and the denominator denotes cochannel interference only. Since $\mathbf{A}_{ij} = \mathbf{0}$ for $i \neq j$, the problem of finding feasible power vectors that satisfy (23) decomposes into N separate problems of the form $((1+\gamma)/\gamma)\mathbf{p}_n = \mathbf{\Delta}_n\mathbf{A}_{nn}(\mathbf{U}_n)\cdot\mathbf{p}_n$ for each channel n, where $\mathbf{\Delta}_n$ is the diagonal matrix that contains reciprocal of useful signal powers of users in channel n, $\mathbf{U}_n$ is the ensemble of beamforming vectors of users in n and $\mathbf{p}_n$ is a $(|\mathcal{U}^{(n)}| \times 1)$ power vector. In OFDMA with orthogonal subcarriers, each user k has different spatial covariance matrix $\mathcal{H}_{n,k}$ for each subcarrier n. Finally in CDMA, codes are non-orthogonal due to cross-correlations and user spatial covariance matrices do not depend on codes, namely it is $\mathcal{H}_{n,k} = \mathcal{H}_k$, for $n = 1, \ldots, N$. For these multiple access schemes, algorithms A and B are executed whenever channel quality varies with time.

A special note should be made for Algorithm C. In CDMA, the algorithm is executed as described previously and the objective is to provide the highest acceptable common SIR to users. In TDMA and OFDMA with orthogonal channels and $\mathbf{F}_{ij} = \mathbf{0}$ for $i \neq j$, a separate problem of the form (31) is solved for each channel n. Algorithm C is modified as follows. We start by assigning all K users in each channel n and we execute Algorithm I for each channel. The outcome is a vector of common SIRs $\boldsymbol{\gamma_c} = (\gamma_{c,1}, \ldots, \gamma_{c,N})$, where $\gamma_{c,n}$ is the resulting common SIR of users in channel n. If $\gamma_{c,n} \geq \gamma$ for all n, desirable SIRs are achieved for cochannel users in all channels and the algorithm terminates. Otherwise some users need to be removed from channels in which the common SIR does not exceed γ. For each user k in such a channel n, let $\gamma_{c,n}(k)$ denote the common SIR of users in channel n after k is removed from n. At each step, we remove the user k from a channel n, so that the resulting $\gamma_{c,n}(k)$ is maximum over possible user removals from channels. User rates are updated after each step of the algorithm and if a user k reaches x_k, it is not considered in further removals. If $\gamma_{c,n} \geq \gamma$ for a channel n at some stage of the algorithm, no further users are removed from this channel. The algorithm terminates when $\gamma_{c,n} \geq \gamma$ for all n.

F. Optimal solution for a special case

We now consider the special case of $K = 2$ users in a channel for $M \geq 2$. The objective is to find the maximum common achievable SIR γ_c^* of users and the beamforming vectors and powers that achieve this SIR. Let $\mathcal{H}_i$, $\mathbf{u}_i$ and p_i be the spatial covariance matrix, beamforming vector and power for user i, $i = 1, 2$. Start with initial beamforming vectors $\mathbf{u}_i^{(0)}$. In the first iteration of Algorithm I, we find $\lambda_{max}^{(1)}$ as a function of $\mathcal{H}_i$ and $\mathbf{u}_i^{(0)}$ and power ratio $\mu^{(1)} = p_2/p_1$ in step 2. In step 3, we find beamforming vectors $\mathbf{u}_1 = \mathbf{u}_{max}(\mathcal{H}_1, \mathcal{H}_2)$ and $\mathbf{u}_2 = \mathbf{u}_{max}(\mathcal{H}_2, \mathcal{H}_1)$. In the second iteration, we have

$$\lambda_{max}^{(2)} = \sqrt{\lambda_{max}(\mathcal{H}_1, \mathcal{H}_2)\lambda_{min}(\mathcal{H}_1, \mathcal{H}_2)} \tag{39}$$

and power ratio $\mu^{(2)} = \sqrt{\lambda_{max}(\mathcal{H}_1, \mathcal{H}_2)/\lambda_{min}(\mathcal{H}_1, \mathcal{H}_2)}$, where $\lambda_{max}(\mathcal{H}_1, \mathcal{H}_2)$ and $\lambda_{min}(\mathcal{H}_1, \mathcal{H}_2)$ are the maximum and minimum generalized eigenvalues of $(\mathcal{H}_1, \mathcal{H}_2)$. These do not change in subsequent iterations. Thus the maximum common SIR is

$$\gamma_c^* = \frac{1}{\sqrt{\lambda_{max}(\mathcal{H}_1, \mathcal{H}_2)\lambda_{min}(\mathcal{H}_1, \mathcal{H}_2)}} \tag{40}$$

with beamforming vectors $\mathbf{u}_1, \mathbf{u}_2$ and power ratio given above.

IV. Simulation Results

A. Simulation setup

We consider a single-cell system with $K = 10$ users that are uniformly distributed in the cell area. The BS uses TDMA, CDMA or OFDMA and is equipped with an antenna array with $M = 4$ elements and $\delta = \lambda/2$. For illustrative reasons, we consider a system with $N = 10$ available channels. Due to single-rate transmission, minimum rate requirements of users are normalized by channel rate and are given in terms of a minimum number of channels. Thus, each user k needs at least $x_k = 3$ channels. The received power decays with distance d from the BS as d^{-4}. For each link corresponding to an antenna and a user receiver, multi-path fading is simulated with a 2-ray model. The angle of the first path, θ_1 is uniformly distributed in $[0, 2\pi]$, while the angle of the second path θ_2 deviates from θ_1 by a random amount, uniformly distributed in $[0, 0.1\pi]$. The complex gain of each path is an independent log-normal random variable with standard deviation $\sigma = 6\,\text{dB}$, which accounts for shadow fading.

B. Comparative results

The primary objective of the simulations is to evaluate and compare the performance of algorithms A, B and C and the different alternatives for beamforming and power control in TDMA, OFDMA or CDMA. It is also desirable to assess the performance benefit of power control in algorithms A and B. Hence, we present results for these algorithms with and without power control (NPC). The performance metrics are the total achievable system rate in terms of number of utilized user channels and the total residual rate, which is defined as the additional required rate, so that users reach their minimum rate requirements. An algorithm is preferable if it results in high system rate and low total residual rate. For CDMA, we assume that code cross-correlation is uniformly distributed in $[0, \rho_{max}^2]$ and we consider the cases of low and high cross-correlation, where $\rho_{max}^2 = 0.02$ and 0.1 respectively. Results are averaged over several random experiments with different channel conditions. The observed fluctuations in plots are due to minimum rate requirements of users. When these are omitted, curves are expected to be smoother.

In figure 4, the total system rate is depicted as a function of SIR threshold γ for OFDMA. A high SIR threshold corresponds to a stringent BER requirement. Algorithm C achieves the best performance for the entire range of values of γ, while algorithm A always performs slightly better than B. Furthermore, power control seems to provide more significant benefits when incorporated in algorithm A. Thus, for moderate values of γ (in the range of $10-15$dB), rate improvements of about $20-25\%$ are achieved by power control in algorithm A, while the corresponding rate benefit of power control in B is only $5 - 10\%$. In addition, the performance of algorithm B with no power control is relatively close (within $5 - 10\%$) to that of A with power control. This seems to suggest that algorithm B with no power control could be adopted in situations where reduced algorithmic complexity is a prerequisite. For large values of γ (e.g., $\gamma > 17$dB), the three of the four alternatives of algorithms A and B result in similar performance. Similar conclusions can be derived for TDMA (figure 5), where algorithm C again yields the highest rate. However, the performance difference between C and the other techniques is smaller than that in OFDMA for a wide range of γ. Another important observation is that algorithm B is slightly better than A.

In figures 6 and 7, performance results are illustrated for CDMA with low and high code cross-correlation. For low cross-correlation, algorithm C yields the highest total rate, while algorithm A results in similar performance. In addition, algorithm A-NPC performs better than B regardless of the use of power control in B, although A-NPC achieved the lowest rate in TDMA and OFDMA. As code cross-correlation increases, algorithms A and C achieve very similar rate and the performance gap between these two and the other algorithms decreases. Finally, in figure 8 we depict the performance of the algorithms in terms of total residual rate for OFDMA. Algorithm C yields much better performance than all other techniques. Minimum rate requirements of users are always satisfied for $\gamma \leq 14$dB and a very small portion of user requirements remain unsatisfied for larger γ. Algorithms A and B result in similar performance.

We also considered the case of 8 antennas but we did not include the corresponding plots due to space limitations. We observed that algorithms A and B for $M = 8$ yield rate only $30 - 35\%$ more than algorithm C with $M = 4$, while algorithm C achieves almost double rate for $M = 8$. This justifies the claim that performance depends both on physical layer methods and channel allocation at the MAC layer. Our results suggest that the SIR balancing algorithm C that involves joint adaptation of beamforming vectors and powers always outperforms greedy algorithms A and B, where the computation of beamforming vectors and powers is decoupled. This difference in performance is evident in OFDMA and TDMA. However, greedy algorithms and SIR balancing algorithms have similar performance in CDMA with non-orthogonal channels. A direct comparison of achievable system rates for different access schemes is not possible, since the allocation is performed on a different resource basis.

V. Discussion

We investigated the impact of SDMA on channel allocation in order to increase system rate and provide QoS to users in the form of minimum rate guarantees. Due to the inherent difficulty in finding the optimal solution, heuristic algorithms must be adopted, which capture desired properties of a good solution. In this paper, we adhered to a unified approach that encompasses TDMA, OFDMA and CDMA and presented three such algorithms for joint channel allocation, beamforming and power control. The first two algorithms use greedy assignment criteria and decouple the operations of beamforming and power control. The third one is based on SIR balancing for user assignment and employs joint beamforming and power control. Performance results indicate that this combination of SIR balancing assignment with joint beamforming and power control yields significantly better performance than other algorithms, especially for access schemes with orthogonal channels.

There exist several directions for future study. In this paper, we considered single-rate transmission in order to illustrate the properties of the algorithms and the interaction between the physical and the MAC layer. The same problem with multi-rate capabilities is an interesting perspective, since spatial separability of users also depends on user rates in a channel. In TDMA and OFDMA, different rates can be provided by adapting modulation level of users on a channel (slot or subcarrier) basis. In CDMA, transmission rate can be varied by adapting the spreading gain of a code or the modulation level of user symbols. The common denominator in all these cases is the inherent tradeoff between high rate and sustainable amount of cochannel interference. For instance, a high modulation level yields high rate per channel but renders spatial separability difficult, since it requires higher SIR so as to maintain a given BER and thus it disallows the formation of large cochannel sets. Furthermore, codes with low spreading gain have higher rate but they usually have higher cross-correlation with other codes, they are associated with lower SIRs and they do not allow large code reuse.

Another interesting direction would be to consider multi-cell systems, in which a user is characterized by different spatial covariance matrix with respect to each BS. BS assignment can balance traffic load, alleviate interference and improve system performance when combined with appropriate beamforming and power control. As a first step, single-channel linear multi-cell systems can be studied, in which a user can be assigned to one of at most three surrounding BSs. The identification of meaningful objectives and heuristics for different multiple access schemes and the incorporation of channel allocation as another dimension to improve performance are some issues that warrant further investigation.

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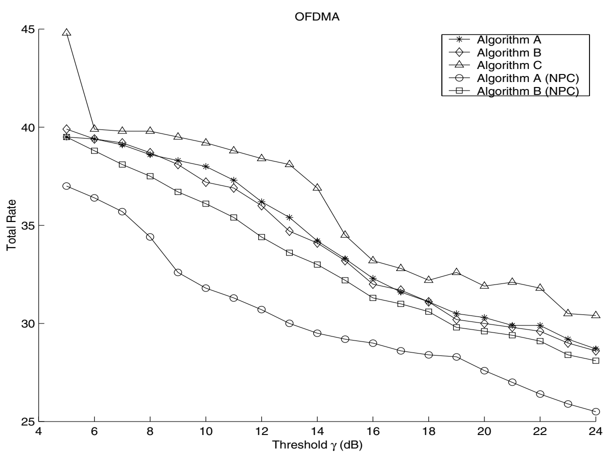
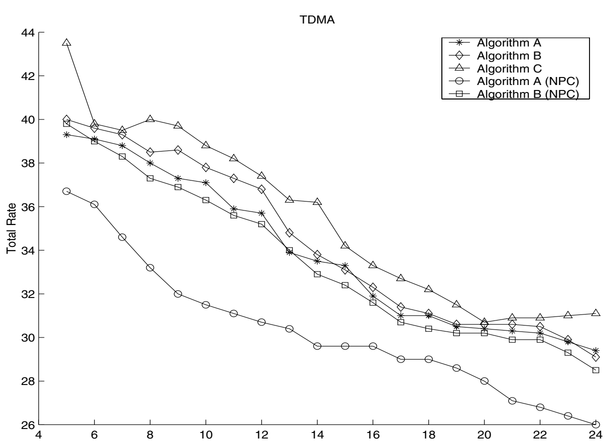


Fig. 4. Total achievable system rate vs. SIR threshold for OFDMA.



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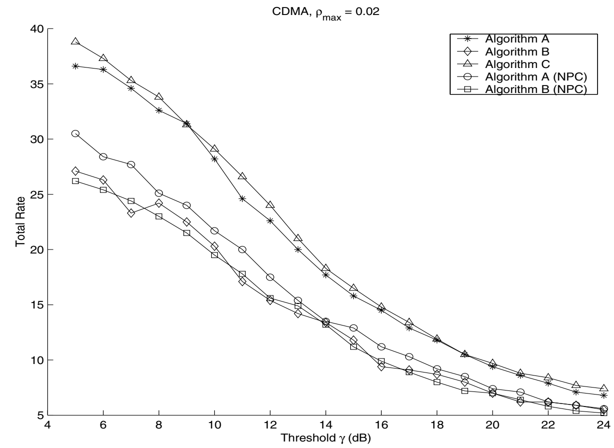

Fig. 6. Total achievable system rate vs. SIR threshold for CDMA with low code cross-correlation.

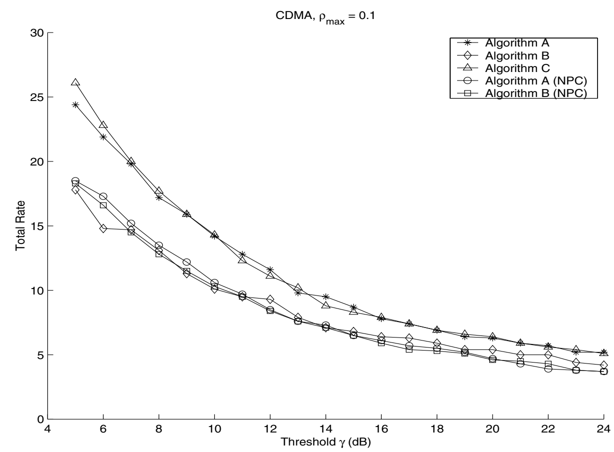

Fig. 7. Total achievable system rate vs. SIR threshold for CDMA with high code cross-correlation.

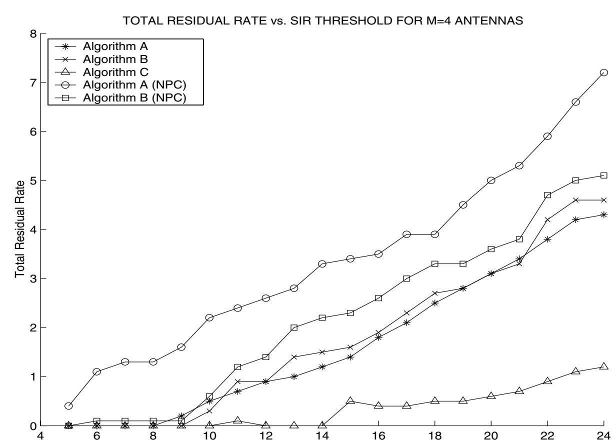

Fig. 8. Total residual rate vs. SIR threshold for OFDMA.