Behavior of Distributed Wavelength Provisioning in Wavelength-Routed Networks with Partial Wavelength Conversion

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Abstract— **Distributed wavelength provisioning is becoming one of the most important technologies for supporting the next-generation wavelength-routed networks. In this paper we analyze the behavior of wavelength-routed networks with partial wavelength conversion capabilities (i.e., where wavelength conversion is available at only a subset of network nodes) when using distributed wavelength provisioning. Simulation results show that the proposed models are highly accurate for different network topologies under various traffic loads.**

I. Introduction

WDM optical networks are widely regarded as the best choice for providing the huge bandwidth required by future networks. To efficiently utilize the bandwidth resources and to eliminate the high cost and bottleneck caused by opto-electronic conversion and processing at intermediate nodes, end-to-end *lightpaths* [1] are usually set up between each pair of source-destination nodes. Lightpaths based WDM networks are generally referred to as *wavelength-routed* networks [2], and the providing of bandwidth by allocating wavelengths to connections via setting up and tearing down lightpaths is termed as *wavelength provisioning*. While current networks mostly have only rather static wavelength provisioning, where lightpath connections are kept unchanged for quite long time (days, weeks, or even years), dynamic provisioning is considered the wave of the future because it can achieve higher network resources utilization, and much better robustness and flexibility [2].

Dynamic wavelength provisioning in WDM networks can be handled in a centralized or distributed way. In *centralized* wavelength provisioning, with information available at a single location, the required lightpaths may be allocated more efficiently as long as optical networks remain relatively small and the traffic is not bursty in nature. To deal with the growth of optical networks and the need for a dynamic allocation of lightpaths under more bursty traffic loads, *distributed* wavelength provisioning has been proposed [3]-[7] and is being standardized in the framework of GMPLS [8].

In either centralized or distributed wavelength provisioning, a lightpath connection request has to be blocked when (1) a route with sufficient free capacity cannot be found from the source to the destination, or (2) a wavelength cannot be found between source and destination though there is free capacity on every hop of the path. The latter one is also known as connection blocking due to *wavelength continuity constraint* [1]. It has been shown that for either centralized or distributed wavelength provisioning schemes, connection blocking due to wavelength continuity constraint could be the dominant part of the overall connection blocking, especially under light traffic load [1], [7].

To lower or even to eliminate the connection blocking due to wavelength continuity constraint, extensive work has been done on wavelength conversion technologies (e.g., [9], [10]). By converting the incoming signal from one wavelength to another, the wavelength continuity constraint can be removed or, at least, be partially relaxed.

Since wavelength conversion is expected to be an expensive technology in foreseeable future [9], [10], *partial wavelength conversion*, i.e., the case where wavelength conversion is available at only a subset of network nodes, becomes an attractive alternative solution (e.g., [11], [12]). It has been shown that in WDM networks with *centralized* wavelength provisioning, partial wavelength conversion could achieve nearly the same performance as that of the full wavelength conversion case [12].

However, in spite of its importance, the analytical study of *distributed* wavelength provisioning in WDM networks with partial wavelength conversion capabilities has not been available so far.

The key behavior metric in either the centralized or the distributed wavelength provisioning is the connection blocking probability, which has been studied analytically for non-conversion or full-conversion cases in a number of previous works [12]-[20]. In [13], an analytical model is proposed based on the assumption that wavelength use on each link is characterized by a fixed probability, independent of other wavelengths and links. As a result, the resulting model has low complexity but does not yield accurate results under wavelength continuity constraint. In [14], the *reduced load approximation* approach [15] with the *state-dependent arrival model* [16] is used in blocking analysis. This model has been shown to be quite accurate for small networks but has a computational complexity growing exponentially with the number of hops. In addition, it is based on the assumption that the set of available wavelengths on adjacent links are independent. This *link independence assumption* is not valid for networks with sparse topologies. In [12], blocking probability is calculated based on the assumption that the load on the i-th hop of a path is only related to the load on $(i-1)$-th hop of it. While this is the first model to consider *link correlation* in blocking analysis, the proposed model is applicable only to uniform traffic situations and regular network topologies. The work in [17] presents an analytical model that provides similar quality results as to [14], but with a much lower computational complexity. In addition, this work proposes a link correlation model applicable to any network topology. Analytical models proposed in [14] and [17] have also been extended to analyze some specific distributed wavelength provisioning schemes [18], [19]. While most studies are based on the link independence assumption or a simplified link correlation model (i.e., link correlation only exists between two adjacent links of a path) in order to keep low computational complexity, an exception is [20], in which a network is decomposed into a set of *path* subsystems. It is claimed that by using this method, higher accuracy can be achieved, though the computational complexity may also be higher.

Some of these analytical models, e.g., the ones proposed in [13], have been successfully extended to analyze *centralized* wavelength provisioning in the networks with partial wavelength conversion (e.g., [12]). Unfortunately, these models cannot be easily extended to analyze *distributed* provisioning in WDM networks with partial wavelength conversion.

In distributed wavelength provisioning, since the information flooding in the networks is usually periodical [3], [4], and propagation delay is unavoidable, *updated, "current" global information* about wavelength availability cannot be guaranteed at any particular place and time in the distributed system. Therefore, there are two different types of connection blocking in distributed systems. The first one is the same as that in centralized provisioning: When wavelength capacity is not available, or when the wavelength continuity constraints (in the networks with no wavelength conversion or partial wavelength conversion) cannot be fulfilled, a connection request has to be blocked. We call such type of blocking as blocking due to *insufficient network capacity*. In addition to that, connection blocking may also occur due to having *outdated global information*. As explained earlier, when a control message reaches a link in order to reserve a wavelength channel on it, it is possible that the capacity that was available when the state information of the link was collected, has in the meantime been reserved by another connection request. We call this type of blocking *outdated information* related, being inherent to virtually all the distributed wavelength provisioning schemes. Partial wavelength conversion, though it cannot eliminate either one of the two types of connection blocking, can be helpful to lower both of them.

In this paper, we analyze the behavior of distributed wavelength provisioning in wavelength-routed networks with partial wavelength conversion capabilities. By using a specific distributed wavelength provisioning scheme as a case study, we analyze both types of connection blocking mentioned above. The effects of partial wavelength conversion are also dealt with in the proposed models. Simulation results show that the analytical models are highly accurate for different network topologies under various traffic loads.

The paper is organized as follows. In Section II, we propose the specific distributed wavelength provisioning scheme which we will use as the case study. The analysis of both blocking due to insufficient network capacity and blocking due to outdated information is presented in Section III, where effects of partial wavelength conversion are also discussed. Numerical results are presented in Section IV. Section V concludes the paper.

II. Distributed Wavelength Provisioning Scheme

The specific distributed wavelength provisioning scheme that we will study in this paper is a straightforward extension of the well-known *destination-initiated reservation* (DIR) method [5]. The DIR method was proposed for the network with no wavelength conversion. In the DIR method, a *connection request* is forwarded from the source to the destination collecting on the way the wavelength availability information along the path. Based on this information, the destination node will select an available wavelength (if such is available along the path) and

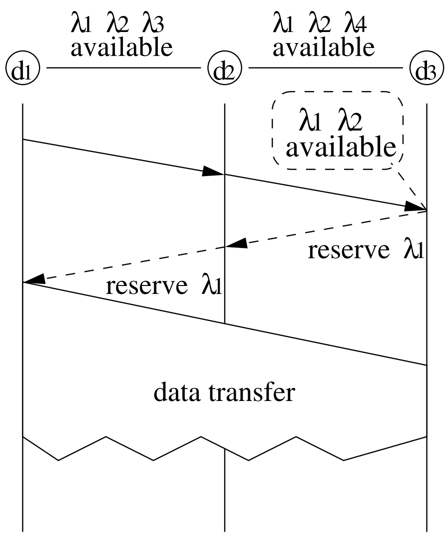

Fig. 1. An example of the DIR method.

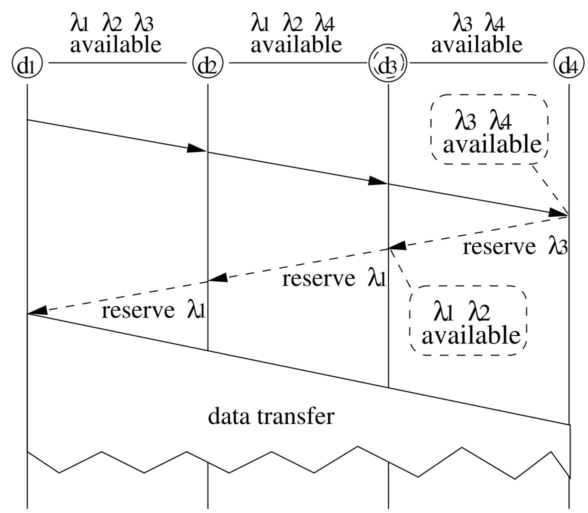

Fig. 2. An example of extended DIR method in the networks with partial wavelength conversion

send a *reservation request* back to the source node to reserve the selected wavelength. Fig. 1 shows an example of the DIR method.

The DIR method could be extended to handle wavelength provisioning in the networks with partial wavelength conversion capabilities. The only difference is that when a reservation request reaches a node with wavelength conversion, if the wavelength the reservation request attempts to reserve has in the meantime been reserved by another reservation request arrived earlier, then another free wavelength could be selected and reserved (if such is available). Different extensions could have slightly different strategies of selecting another free wavelength. To simplify the description of the specific extension that we will study in this paper, we make some definitions as follows:

- A route from the source to the destination can be composed of one or several *segments*. The two end nodes of each segment can only be the source node or the destination node, or the node with wavelength conversion. Wavelength conversion is not available on *any* intermediate node of a segment.
- Among the two connected links or segments in a route, we call the one closer to the destination node as the *downstream* one, and the one closer to the source node as the *upstream* one.
- Among the two end nodes of a link or a segment, we call the one closer to the destination as the *right-hand* node, and the other end node the *left-hand* node.

Based on these definitions, we define the specific distributed scheme as follows: Between each pair of source-destination nodes, we always use the fixed shortest-path routing between them. At the destination node, a wavelength that is available in the last segment (according to the information collected by the connection request) will be *randomly* selected for reservation (if such are available). When the reservation request reaches the right-hand node of a segment, if the wavelength that had been reserved in the downstream segment is available in both the upstream link (based on local, updated wavelength availability information) and the upstream segment (according to the information collected by the connection request), then the reservation request will try to reserve the same wavelength in the upstream segment; otherwise, another wavelength that is available in the upstream segment (according to the information collected by the connection request) will be *randomly* selected and reserved. If no other wavelength is available in the upstream segment (according to the information collected by the connection request), the reservation request will be blocked even if there are some available wavelengths in the upstream link. An example of this specific distributed scheme is shown in Fig. 2.

III. Analytical Model

A. Framework

There are two types of connection blocking when the proposed distributed provisioning scheme is used:

- Blocking in the *forward direction* (i.e., the direction from the source to the destination), due to insufficient network capacity. This type of blocking is also termed *forward blocking*.
- Blocking in the *backward direction* (i.e., the direction from the destination back to the source), caused by out-

dated information. This type of blocking is also termed *backward blocking*.

To simplify our analysis, we make the following assumptions: The network is composed of J links connected in an arbitrary topology where each link is composed of C wavelength channels. Wavelength conversion is available only on a certain given set of nodes. The connection requests between each pair of source-destination nodes arrive from a Poisson process with an arrival rate λ_R, where R denotes the fixed route between the two nodes.

In this paper, we let the *link state* be the state of a link when a connection request reaches the right-hand node of the link [1]. A wavelength channel can be in one of the following three states: (1) free; (2) reserved, yet with no data transmission; and (3) occupied by data transmission. We shall say that in the state (3), the wavelength channel is *busy*; otherwise, it is *idle*.

Let X_j ($j = 1, 2, \cdots, J$) be the random variable representing the number of idle wavelength channels on link j. Let

$$q_j(m) = \Pr\{X_j = m\}, \qquad m = 0, 1, \cdots C \tag{1}$$

be the probability that there are exactly m idle wavelength channels on link j. Following [16] we assume that all X_j's are mutually independent, then we have

$$q(\mathbf{m}) = \prod_{j=1}^{J} q_j(m_j) \tag{2}$$

where

$$\mathbf{m} = (m_1, m_2, \cdots m_J)$$

We further assume that when there are m idle wavelength channels on link j, the inter-arrival time of connection requests is exponentially distributed with a parameter $\lambda_j(m)$. The holding time for each connection follows exponential distribution with the same rate μ. Therefore, we have

$$\begin{aligned} q_j(m) &= \tfrac{C(C-1)\cdots(C-m+1)}{\lambda_j(1)\lambda_j(2)\cdots\lambda_j(m)} \cdot \mu^m \cdot q_j(0), \\ & m = 1, 2, \cdots C \end{aligned} \tag{3}$$

where

$$q_j(0) = \left[1 + \sum_{m=1}^{C} \frac{C(C-1)\cdots(C-m+1)}{\lambda_j(1)\lambda_j(2)\cdots\lambda_j(m)} \cdot \mu^m\right]^{-1} \tag{4}$$

Finally, the framework for calculating the steady state probability $q(\mathbf{m})$ can be summarized as follows:

[1] The reason we make this definition is: Due to the propagation delay, the state of a link can be changed during the period of time when a connection request is moving from the left-hand node to the right-hand node of this link. Therefore, the state information provided by the right-hand node is more updated.

Calculating the Connection Blocking: Framework

1) Initiate $\lambda_j(m), j = 1, 2, \cdots, J$ as follows:

$$\lambda_j(m) = \begin{cases} \sum_{R:j\in R} \lambda_R, & m = 1, 2, \cdots, C \\ 0, & m = 0 \end{cases} \tag{5}$$

2) Calculate $q(\mathbf{m})$ through equations (3)-(4).
3) Calculate the blocking probability of R as

$$\begin{aligned} B_R &= 1 - V_R \\ &= 1 - V_R^F \times V_R^B \\ &= 1 - (1 - B_R^F) \times (1 - B_R^B) \end{aligned} \tag{6}$$

where V_R denotes the probability that a reservation is successful along the route R, V_R^F denotes the probability that a reservation is successful in the forward direction, and V_R^B denotes the conditional probability that a reservation is successful in the backward direction given that it is successful in the forward direction. If for every route R, B_R has been convergent, then stop; otherwise, go to step 4.
4) Calculate $\lambda_j(m), j = 1, 2, \cdots, J$ as follows:

$$\lambda_j(m) = \sum_{R:j\in R} \lambda_{R,j}(m) \triangleq \sum_{R:j\in R} \lambda_R \cdot V_{R|X_j=m} \tag{7}$$

where $\lambda_{R,j}(m)$ denotes the arrival rate of those connection requests for route R which are finally successfully accepted, given that the state of link j is m. Go to step 2.

In step 3, we consider the blocking in both the forward and backward directions as shown in equation (6). In the following subsections, we will discuss the calculation of B_R^F, B_R^B and $\lambda_j(m)$, respectively.

B. Blocking due to Insufficient Network Capacity

Connection requests can be blocked in the forward direction due to insufficient network capacity. With partial wavelength conversion, wavelength continuity constraint can be partially relaxed and therefore forward blocking probability can be lowered. The main idea of the blocking analysis basically comes from [14], [17]: It is based on a link correlation model where the state dependent model is used to describe the link state. However, we take the influence of propagation delay of management messages into consideration. Specifically, due to the propagation delay of reservation request in the *backward* direction, some wavelength channels are reserved for a short period of time before they are actually occupied by data transmission. Such type of reservation could consume some network capacity and make the blocking probability in the *forward* direction slightly higher. This type of influence could be significant when under bursty traffic load. Further improvement in analysis accuracy is achieved by modifying the model proposed in [17] to better analyze the state dependent arrival rate of traffic requests, as will be explained later in Section III-C. To take the effects of partial wavelength conversion into consideration, we analyze the blocking probability in each *segment* because a connection is successful if and only if it is successful in every segment of the route. Below we present the detailed analysis.

As we have mentioned, a connection request can successfully reach the destination node if and only if in *every* segment of the route there is at least one available wavelength. Here we assume that the probabilities of successful in each segment are independent with each other. Therefore,

$$V_R^F = \prod_{s=1}^{S_R} V_s^F \tag{8}$$

where S_R denotes the number of segments in the route R. Next we analyze the blocking probability in each segment. The analysis method is similar to that for a route in the networks with no wavelength conversion. The detailed analytical model is as follows:

Let $h_{i,s}$ denote the probability that a given set of i wavelength channels are free in segment s of route R at the moment when the connection request reaches the right-hand node of the segment. Then

$$V_s^F = \sum_{i=1}^{C} (-1)^{i+1} \binom{C}{i} h_{i,s} \tag{9}$$

To simplify the description, we denote j-th link of segment s as link j and the $(j-1)$-th link (when $j > 1$) the link j'. Let $Y_{k,j}(t)$ denote the state (busy or idle) of channel k on link j at time t, and t_j denote the propagation delay on link j. To simplify the analysis, we make the following assumptions [17]:

1) All wavelength channels are statistically identical. This assumption is reasonable since we are using random wavelength assignment.
2) $Y_{k_1,j}(t_j)$ is independent of $Y_{k_2,j'}(0)$ ($k_1 \neq k_2$) given that $Y_{k_2,j}(t_j)$ or $Y_{k_1,j'}(0)$ is known.
3) $Y_{k,j}(t_j)$ is independent of $Y_{k,j^*}(t)$ ($j^* \neq j, j', \forall t$) given that $Y_{k,j'}(0)$ is known.

Based on the assumptions, we have

$$h_{i,s} = \begin{cases} h_{i,1}, & \text{if } L_s = 1 \\ h_{i,1} \cdot \prod_{j=2}^{L_s} h_{i,j|i,j'}(t_j), & \text{otherwise} \end{cases} \tag{10}$$

where L_s denotes the hop length of segment s, and $h_{i,j|i,j'}(t_j)$ denotes the conditional probability that a given set of i wavelength channels are free on link j given that t_j time slots ago they were free on link j'. Therefore,

$$\begin{cases} h_{i,1} &= g_{i,1} \times f_{i,1} \\ h_{i,j|i,j'}(t_j) &= g_{i,j|i,j'}(t_j) \times f_{i,j|i,j'}(t_j) \end{cases} \tag{11}$$

where

- $g_{i,j}$ denotes the steady state probability that a given set of i wavelength channels are idle on link j.
- $f_{i,j}$ denotes the conditional probability that a given set of i channels are free on link j given that these i channels are idle.
- $g_{i,j|i,j'}(t_j)$ denotes the conditional probability that a given set of i wavelength channels are idle on link j given that t_j time slots ago they were idle on link j'.
- $f_{i,j|i,j'}(t_j)$ denotes the conditional probability that a given set of i wavelength channels are free on link j given that these i channels are idle and t_j time slots ago they were free on link j'.

Below we will discuss the calculations of $g_{i,j}$, $f_{i,j}$, $g_{i,j|i,j'}(t_j)$ and $f_{i,j|i,j'}(t_j)$, respectively.

Calculating $g_{i,j}$ and $g_{i,j|i,j'}(t_j)$

From the definition of $q_j(m)$, we have

$$g_{i,j} = \sum_{m=i}^{C} q_j(m) \cdot g_{i,j|X_j=m} \tag{12}$$

where

$$g_{i,j|X_j=m} = \frac{\binom{m}{i}}{\binom{C}{i}} = \prod_{k=1}^{i} \frac{m-k+1}{C-k+1} \tag{13}$$

Let $F_{k,j}(t_j)$ denote the event that the k-th channel on link j is idle at time t_j and $\overline{F_{k,j}}(t_j)$ denote the opposite event. Based on the assumptions before equation (10), we have

$$\begin{aligned} & g_{i,j|i,j'}(t_j) = \\ & \quad \Pr\{F_{i,j}(t_j) | F_{i-1,j}(t_j), \cdots, F_{1,j}(t_j); F_{i,j'}(0)\} \\ & \quad \times \Pr\{F_{i-1,j}(t_j) | F_{i-2,j}(t_j), \cdots, F_{1,j}(t_j); F_{i-1,j'}(0)\} \\ & \quad \times \cdots \\ & \quad \times \Pr\{F_{2,j}(t_j) | F_{1,j}(t_j); F_{2,j'}(0)\} \\ & \quad \times \Pr\{F_{1,j}(t_j) | F_{1,j'}(0)\} \end{aligned} \tag{14}$$

From the link correlation model,

$$\begin{aligned} & \Pr\{F_{i,j}(t_j) | F_{i-1,j}(t_j), \cdots, F_{1,j}(t_j); F_{i,j'}(0)\} \\ & = \left[1 + \gamma_{j',j}(t_j) \times \left(\frac{1}{\eta_{i,j}} - 1\right)\right]^{-1} \end{aligned} \tag{15}$$

where $\eta_{i,j}$ denotes conditional probability that channel i is idle on link j given that all the channel 1 through channel $(i-1)$ are idle, i.e.,

$$\eta_{i,j} = \begin{cases} g_{i,j}, & i = 1 \\ \frac{g_{i,j}}{g_{i-1,j}}, & i > 1 \end{cases} \tag{16}$$

and

$$\gamma_{j',j}(t_j) = \frac{\Pr\{\overline{F_{i,j}}(t_j)|F_{i,j'}(0)\}}{\Pr\{F_{i,j}(t_j)|F_{i,j'}(0)\}} = \frac{\lambda_{j,\overline{j'}}}{\lambda_j} \tag{17}$$

In equation (17), λ_j denotes the average rate of the connection requests passing through link j and are finally accepted; and $\lambda_{j,\overline{j'}}$ denotes the average arrival rate of the connection requests passing through link j but not passing through j' and are finally accepted. For more discussions on equations (15)-(17), please refer to [17].

To summarize, we have

$$g_{i,j|i,j'}(t_j) = \prod_{k=1}^{i} \left[1 + \gamma_{j',j}(t_j) \times \left(\frac{1}{\eta_{k,j}} - 1\right)\right]^{-1} \tag{18}$$

Calculating $f_{i,j}$ and $f_{i,j|i,j'}(t_j)$

Variable $f_{i,j}$ denotes the probability that a given set of i wavelength channels are free on link j given that these i wavelength channels are idle. This conditional probability measures the influence of propagation delay. From the moment a channel is reserved to the moment it becomes busy, the length of the time interval equals to

$$\tau_R(j) = 2 \times \sum_{l=1}^{j} t_l \tag{19}$$

which is the round-trip propagation delay from the source node of route R to the right-hand node of link j. Therefore, $f_{i,j}$ can be calculated as follows:

$$f_{i,j} = \sum_{m=i}^{C} q_{j|i}(m) \prod_{R:j\in R} \left(1 - \left(1 - e^{-\lambda_{R,j}(m)\tau_R(j)}\right) \times \frac{i}{m}\right) \tag{20}$$

where $q_{j|i}(m)$ denotes the probability that m channels are idle on link j given that a specific set of i channels ($i \le m$) are idle on this link, i.e.,

$$q_{j|i}(m) = q_j(m) \times \frac{g_{i,j|X_j=m}}{g_{i,j}} \tag{21}$$

The basic idea for calculating $f_{i,j|i,j'}(t_j)$ is nearly the same as that for calculating $f_{i,j}$. The only difference is: If the reservation request also passes through link j' and at time t the channel on link j is reserved but not busy, then the reservation request must have arrived the right-hand node of link j within the time interval $(t - 2t_j, t)$; otherwise, the same wavelength on link j' should have been reserved at time $t - t_j$. Therefore, we define that for any route R passing through link j,

$$\tau_R(j, j') = \begin{cases} \tau_R(j) & j' \notin R \\ 2 \times t_j & j' \in R \end{cases} \tag{22}$$

Then $f_{i,j|i,j'}(t_j)$ can be calculated by using equation (20) where $\tau_R(j)$ is replaced by $\tau_R(j, j')$.

C. Blocking due to Outdated Information

Connection blocking could happen in the backward direction due to outdated information. Specifically, such blocking will happen if and only if we have several reservation requests competing for a same wavelength channel. The detailed analysis is as follows.

Similar to that in the forward blocking analysis, a reservation request is successful if and only if it is successful in *every* segment. Therefore, with the segmentation independence assumption in Section III-B, the probability that a reservation is successful in the backward direction can be derived as

$$V_R^B = \prod_{s=1}^{S_R} V_s^B \tag{23}$$

However, unlike that in the forward blocking analysis, the backward blocking analysis of a segment does not resemble the backward blocking analysis of a route in the networks with no wavelength conversion: In a network with no wavelength conversion, a reservation request will never be blocked at the destination node (i.e., the right-hand node of the route) if, according to the information collected by the connection request, a wavelength is available along the whole route. However, a reservation request can be blocked at the right-hand node of a segment even if a wavelength was available along the whole segment according to the information collected by the connection request. We present the detailed analytical model as follows.

We observe that if a reservation request for route R is blocked on link j, then one or more interfering reservation requests must have arrived *after* the connection request for R passed link j. In other words, the interfering reservation request must have arrived in the past $t_R(j)$ time slots, where $t_R(j)$ denotes the round-trip propagation delay between the right-hand node of link j and the destination of route R, i.e.,

$$t_R(j) = 2 \times \sum_{l=j+1}^{L_R} t_l \tag{24}$$

where L_R represents the number of hops in route R.

On the rightmost link of a segment (i.e., $j = L_s$), the reservation request for route R will be blocked if and only if all the free wavelengths in this link were reserved during the past $t_R(j)$ time slots. On the other hand, if the reservation request gets blocked on another link j ($j < L_s$) of the segment, the interfering reservation request cannot have gone through the $(j+1)$-th link of route R (denoted as j''); otherwise, the reservation request for route R should have been blocked on link j''. Based on these observations, we have

$$V_s^B = \begin{cases} w_s(t_R(j)), & L_s = 1 \\ w_s(t_R(j)) \times \prod_{j=1}^{L_s-1} w_{j,j''}(t_R(j)) & L_s > 1 \end{cases} \tag{25}$$

where $w_s(t_R(j))$ denotes the probability that the reservation request for route R is *not* blocked at the right-hand node of segment s, and $w_{j,j''}(t_R(j))$ denotes the conditional probability that no interfering reservation requests has arrived link j within the past $t_R(j)$ time slots and reserved the same wavelength, given that j'' is not on the route of that interfering reservation request.

Calculating $w_s(t_R(j))$

From the definition of $w_s(t_R(j))$, we have

$$\begin{aligned} w_s(t_R(j)) = 1 - \textstyle\sum_{m=1}^{C} q_j(m) \\ \left[\textstyle\sum_{n=1}^{m} v_{s|X_j=m}(n) \times \left(1 - w_{s|m,n}(t_R(j))\right)\right] \end{aligned} \tag{26}$$

where $v_{s|X_j=m}(n)$ denotes the conditional probability that there are n free wavelengths along the segment s given that there are m idle wavelength channels on link j, and $w_{s|m,n}(t_R(j))$ denotes the conditional probability that the reservation request of R is *not* blocked at the right-hand node of segment given that $t_R(j)$ time slots ago there were n free wavelengths along the segment s and m idle wavelengths on the rightmost link of the segment. Therefore,

$$v_{s|X_j=m}(n) = \binom{C}{n} \cdot \sum_{i=n}^{m} (-1)^{n+i} \binom{C-n}{i-n} h_{i,s|X_j=m} \tag{27}$$

where $h_{i,s|X_j=m}$ denotes the conditional probability that a given set of i channels are free in segment s at the moment when the connection request reaches the right-hand node of segment s, given that m wavelength channels are idle on link j. To calculate $h_{i,s|X_j=m}$, we only need to slightly modify equations (10) and (11) to take the additional condition $X_j = m$ into consideration. In other words, we need to calculate four probabilities: $g_{i,j|X_j=m}$, $g_{i,j|i,j';X_j=m}(t_j)$, $f_{i,j|X_j=m}$, and $f_{i,j|i,j';X_j=m}(t_j)$. Since we already got $g_{i,j|X_j=m}$ in (13), below we will consider the other three probabilities:

$$\begin{cases} g_{i,j|i,j';X_j=m}(t_j) = \prod_{k=1}^{i} \left[1 + \gamma_{j',j}(t_j) \cdot \frac{C-m}{m-k+1}\right]^{-1} \\ f_{i,j|X_j=m} = \prod_{R:j\in R} \left(1 - \left(1 - e^{-\lambda_{R,j}(m)\tau_R(j)}\right) \cdot \frac{i}{m}\right) \\ f_{i,j|i,j';X_j=m}(t_j) = \\ \quad \prod_{R:j\in R} \left(1 - (1 - e^{-\lambda_{R,j}(m)\tau_R(j,j')}) \cdot \frac{i}{m}\right) \end{cases} \tag{28}$$

The first equation in (28) is nearly the same as equation (18) with only one slight difference. That is, there is an additional condition that $X_j = m$, which leads to a more accurate correlation model. With this additional condition, we have from the definition in (16) that

$$\eta_{k,j|X_j=m} = \frac{m-k+1}{C-k+1} \tag{29}$$

Therefore,

$$\frac{1}{\eta_{k,j;X_j=m}} - 1 = \frac{C-m}{m-k+1} \tag{30}$$

To calculate $w_{s|m,n}(t_R(j))$, we let $u_{i,s|m}(t_R(j))$ denote the conditional probability that a given set of i wavelengths are free on the rightmost link of segment s when reservation request for R arrives, given that $t_R(j)$ time slot ago m wavelengths were idle on this link and n ($n \leq m$) wavelengths were free along the segment. Therefore, we have

$$w_{s|m,n}(t_R(j)) = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} u_{i,s|m}(t_R(j)) \tag{31}$$

and

$$\begin{aligned} u_{i,s|m}(t_R(j)) = \\ \textstyle\prod_{R':j\in R'} \left(1 - \left(1 - e^{-\lambda_{R',j}(m)t_R(j)}\right) \times \frac{i}{m}\right) \end{aligned} \tag{32}$$

where R' denotes the route of any interfering reservation request.

Calculating $w_{j,j''}(t_R(j))$

Similar to equation (26), we have

$$w_{j,j''}(t_R(j)) = 1 - \sum_{m=1}^{C} q_j(m) \left(1 - w_{j,j''|X_j=m}(t_R(j))\right) \tag{33}$$

where $w_{j,j''|X_j=m}(t_R(j))$ denotes the conditional probability that the reservation request of R is *not* blocked at link j given that $t_R(j)$ time slots ago there were m idle wavelengths on link j, and j'' is not on the route of any interfering reservation request. Therefore,

$$\begin{aligned} w_{j,j''|X_j=m}(t_R(j)) = \\ \textstyle\prod_{R':j\in R'} \left(1 - \left(1 - e^{-\lambda_{R',j}(m)t_R(j)}\right) \times \frac{1}{m}\right) \end{aligned} \tag{34}$$

where R' still denotes the route of any interfering reservation request.

D. State Dependent Arrival Rate

To complete the calculation of the overall connection blocking probability for the distributed scheme, as described in step 4 of the Framework in Section III-A, it remains to obtain the state dependent arrival rate $\lambda_j(m)$. From equation (7), we see that in order to obtain $\lambda_j(m)$, we need to calculate $V_{R|X_j=m}$.

Similar to equation (6), we have

$$V_{R|X_j=m} = V^F_{R|X_j=m} \times V^B_{R|X_j=m} \tag{35}$$

where $V^F_{R|X_j=m}$ and $V^B_{R|X_j=m}$ are two conditional probabilities that need to be calculated first.

Calculating $V^F_{R|X_j=m}$

$V^F_{R|X_j=m}$ can be calculated as

$$V^F_{R|X_j=m} = V^F_R \times \frac{V^F_{s|X_j=m}}{V^F_s}, \qquad j \in s \tag{36}$$

where

$$V^F_{s|X_j=m} = \sum_{i=1}^{m} (-1)^{i+1} \binom{C}{i} h_{i,s|X_j=m} \tag{37}$$

which resembles equation (9).

Calculating $V^B_{R|X_j=m}$

$V^B_{R|X_j=m}$ can be calculated as

$$V^B_{R|X_j=m} = V^B_R \times \frac{V^B_{s|X_j=m}}{V^B_s}, \qquad j \in s \tag{38}$$

where $V^B_{s|X_j=m}$ can be calculated by using equations (25) with $w_s(t_R(j))$ and $w_{j,j''}(t_R(j))$ be replaced by $w_{s|X_j=m}(t_R(j))$ and $w_{j,j''|X_j=m}(t_R(j))$ respectively. We have

$$\begin{aligned} &w_{s|X_j=m}(t_R(j)) = \\ &\quad \textstyle\sum_{n=1}^{m} v_{s|X_j=m}(n) \times \big(1 - w_{s|m,n}(t_R(j))\big) \end{aligned} \tag{39}$$

and the calculation of $w_{j,j''|X_j=m}(t_R(j))$ can be found in equation (34).

IV. Numerical Results

To evaluate the accuracy of the proposed analytical models, we compare the analysis results to the simulation results on the PacNet (shown in Fig. 3 where the numbers next to the links denote the physical length in tens of kilometers). In all our simulations, unless otherwise specified, we assume that (1) each link is composed of two directional fibers of opposite directions with eight wavelength channels per fiber; (2) the connection requests arrive from a Poisson process with exponentially distributed duration where the average duration equals to 100ms; (3) the traffic pattern is uniform, i.e., the arrival rate of the connection requests between each pair of source-destination nodes is a constant; (4) the fixed shortest path routing is used between each pair of source-destination nodes; and (5) wavelength conversion is available on a randomly selected set of nodes (We assume that wavelength conversion is available on nodes 3, 4, 9 and 11. We have also tried many other different cases where wavelength conversion is available on different set of nodes. The accuracy of the proposed analytical models has never been affected.). In all the figures for simulation results, we let the traffic load measured in Erlang on x-axis denote the average traffic load sourced from every node on every wavelength.

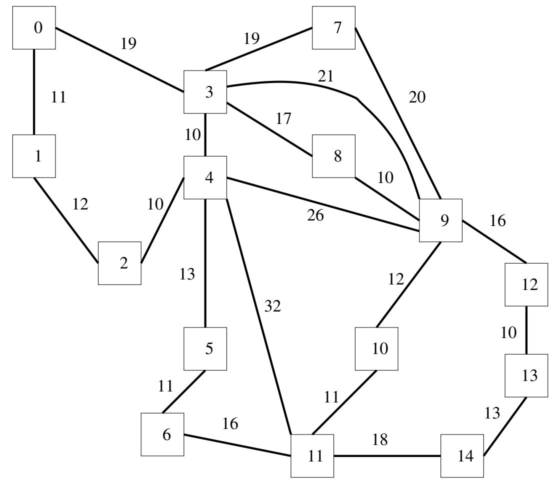

Fig. 3. Network Topology of the PacNet.

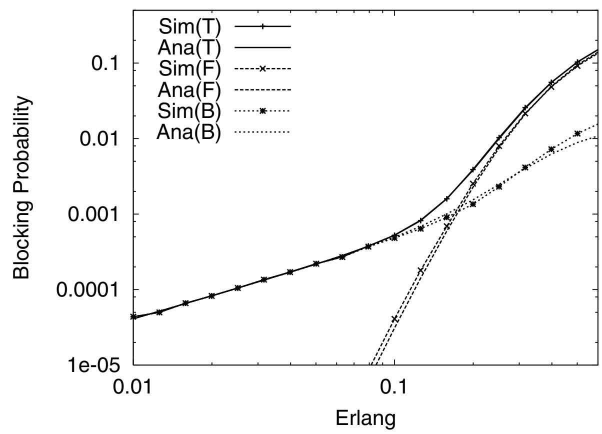

Fig. 4. Blocking probabilities in the PacNet with partial wavelength conversion.

The analysis results of blocking probabilities in both the forward and the backward directions are presented in Fig. 4, which show a very good match with simulation results. In addition, we observe that

- Under light traffic load, the blocking mainly takes place

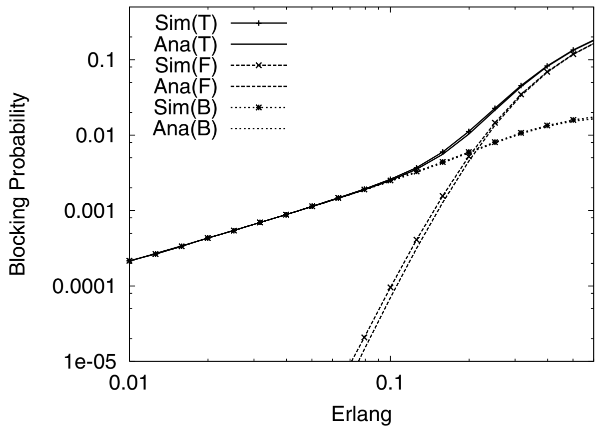

Fig. 5. Blocking probabilities in the PacNet with no wavelength conversion.

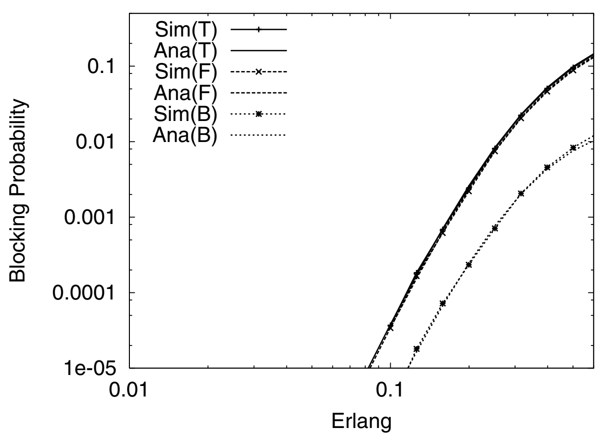

Fig. 6. Blocking probabilities in the PacNet with full wavelength conversion.

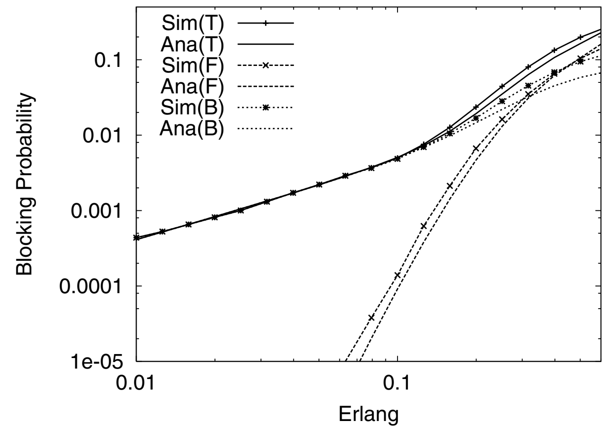

Fig. 7. Blocking probabilities in the PacNet with partial wavelength conversion. Average duration of each connection is 10ms.

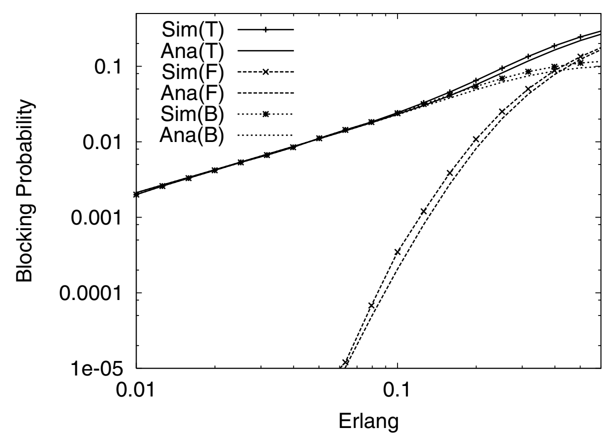

Fig. 8. Blocking probabilities in the PacNet with no wavelength conversion. Average duration of each connection is 10ms.

in backward direction, caused by outdated information; whereas under heavy traffic load, the blocking occurs mainly in forward direction, due to insufficient network capacity. This is the same observation in both the case with the partial wavelength conversion and the case with no wavelength conversion (see Fig. 5).

- Partial wavelength conversion could significantly lower the blocking probability, especially when under light traffic loads (by comparing Fig. 4 and Fig. 5). The main reason is that by using partial wavelength conversion, backward blocking due to outdated information can be significantly lowered.

Fig. 4 also demonstrates that the proposed analytical models are highly accurate.

The analysis results for the special case when we have full wavelength conversion are presented in Fig. 6. We observe that with wavelength conversion on every node to resolve the wavelength conflict, backward blocking can never become a dominant part of connection blocking under either light or heavy traffic load. For this special case, the proposed analytical models can achieve very high accuracy.

Fig. 7 deals with connection blocking when the traffic request arrival rate is higher and the average duration is shorter (10ms), i.e., when the traffic is more bursty. We find that in this case, the blocking probability in the backward direction is significantly higher compared to the case in Fig. 4. This is mainly because that under more bursty traffic load, the blocking probability caused by outdated information is significantly higher. Another reason is that, as we have mentioned in Section III-B, due to the propagation delay of reservation request, some wavelength channels are reserved for a short period of time before they are actually occupied by data transmission. Such type of reservation could consume some network capacity and make the blocking probability in both the forward and the backward

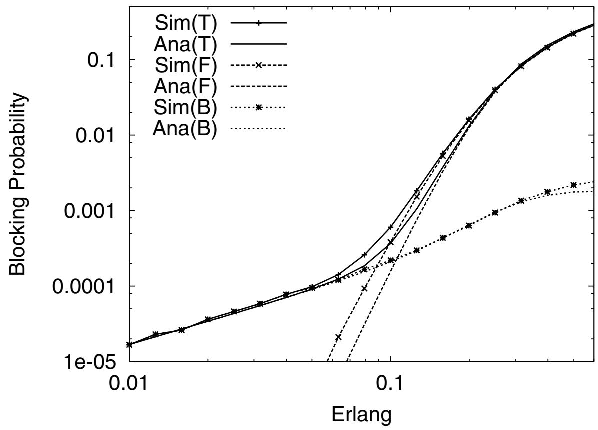

Fig. 9. Blocking probabilities in the 12-node optical ring with partial wavelength conversion

directions higher. The type of influence becomes quite significant when under highly bursty traffic load. We observe that for this case, our analytical models can still achieve highly accurate results. In addition, by comparing Fig. 7 (the case with partial wavelength conversion) with Fig. 8 (the case with no wavelength conversion), we observe that partial wavelength conversion can still significantly lower the blocking probability of the networks when under highly bursty traffic load.

Finally, the performance of the proposed analytical models on the optical ring is presented in Fig.9. We assume that the ring has 12 nodes while each link is of 10 km length. Four nodes with wavelength conversion are evenly distributed along the ring. We observe that, due to the very high correlation between different lightpaths, the analysis results become less accurate compared to those in the PacNet (but still acceptable). In fact, this is also the case in most of the previous studies (e.g., [17]). To get more accurate results, it is widely believed that more complicated models have to be used, which in our case means that the assumptions we made before equation (10) shall be somewhat released. However, how to keep the complexity of computation at a reasonably low level when releasing these assumptions is basically still an open problem.

V. Conclusion

This paper, to the best of authors' knowledge, provided the first analysis of the behavior of distributed wavelength provisioning networks with partial wavelength conversion capabilities. By using a specific extension of the DIR method as a case study, we analyzed system performance under connection blocking occurring due to insufficient network capacity as well as connection blocking caused by outdated information. By introducing the segment concept and analyzing the blocking probability on each segment, effects of partial wavelength conversion could be studied. Simulation results show that the proposed models are highly accurate for different network topologies under various traffic loads.

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